Exercise 3

1 Dual decomposition and dynamic programming

In this exercise we will implement dual decomposition with subgradient descent (See for example: MRF energy minimization and beyond via dual decomposition, Komodakis et al.) for disparity optimization in stereo image pairs.

We will use the cost volume from exercise 1 to solve the following optimization problem:

$$E(D) = \sum_p C(D_p) + \sum_{q \in N_p} P_1 T[|D_p - D_q| == 1] + \sum_{q \in N_p} P_2 1T[|D_p - D_q| > 1]$$

In this optimization problem $C(D_p)$ is the cost of disparity $D_p$ at pixel $p$, i.e. the entry from the cost volume. $N_p$ is the neighborhood of pixel $p$. $P_1$ is a cost for a disparity jump of 1 between neighboring pixels ($T[|D_p - D_q|] == 1$). $P_2$ is a cost for a disparity jump larger than 1 between neighboring pixels.

This NP-hard optimization problem is now decomposed into two sub-problems: one problem consists only of the rows of the image and the pairwise interactions along the rows and the other problem consists of the columns of the image and the pairwise interactions ($P_1, P_2$) along the columns.

We call the cost volume for the row problem $C_r$ and the cost volume for the column problem $C_c$. We now can solve the optimization problem $E(C_c)$ ($E(C_r)$) easily with a dynamic programming approach since it only consists of pairwise interactions along single columns (rows). The solution to the optimization problem $E(C_c)$ ($E(C_r)$) is stored in an indicator volume $V_c$ ($V_r$) that contains a single 1 for each pixel at the best disparity for that pixel.

The subgradient update step for the two cost volumes is then

$$C_c = C_c + \alpha(V_c - V_r)$$
$$C_r = C_r + \alpha(V_r - V_c)$$

the updated cost volumes are then used to solve the dynamic programming problems again and again. The process converges when $V_c = V_r$.

- Implement the dynamic programming and the subgradient update. Good values for the parameters are $P1 = 10$, $P2 = 100$ and $\alpha = 1$.
- Plot the number of disagreements between $V_c$ and $V_r$ for each iteration.
- Visualize the disagreements between $V_c$ and $V_r$ in each iteration.
- Evaluate the Energy of $V_c$ and $V_r$ in each iteration.
- Evaluate the duality gap in each iteration.