Exercise 1

Deadline: 26.10.2018
Format: You may hand in the exercises individually or with a colleague. Please send your solutions as pdf/html to *cvflecture@gmail.com*. To make it easier for me please use your last names as filenames and "[CVF18]" in the email subject line.

Personal Information: For the first exercise please state your:

- Full name
- Pursued degree (e.g. Master in Physics)
- Immatrikulationsnummer
- Email

Recap from the lecture

Given observations $\{(l_i, o_i) \mid i = 1, \ldots, n\}$ we define the non-parametric estimate

$$f(l, r) = \sum_i w_{\text{spatial}}(l - l_i)\rho_{\text{range}}(r - o_i) \tag{1}$$

$$\hat{\mu}(l) = \arg\min_r f(l, r), \tag{2}$$

where possible choices for

- w_{spatial} are
$$\{\text{uniform}, \text{Gaussian}, \text{truncated Gaussian}, \text{Box}, \text{Tukey}, \text{B-Spline}\} \tag{3}$$
- ρ_{range} are
$$\{\text{-Gaussian}, \text{Tukey}, \text{Huber}, \text{l1}, \text{l2}\} \tag{4}$$

Box_δ (in the following exercise) is defined as $w_{\text{Box}}(l) = \begin{cases} 1 & \text{for } |l| < \delta, \\ 0 & \text{otherwise.} \end{cases}$

1 Nonlinear Filtering

1.1 Robust non-parametric estimators (4 points)

To get a more intuitive understanding of w_{spatial} and ρ_{range} let us consider a specific example of observations in Figure 1. Each image shows the same set of observations (white dots) and estimates $f(l, r)$ obtained using different choices of w_{spatial} and ρ_{range}.

Task:

- Find out which
$$w_{\text{spatial}} \in \{\text{Box}_{\delta=2}, \text{Box}_{\delta=6}, \text{Gaussian}_{\sigma=2}, \text{Gaussian}_{\sigma=10}, \text{Gaussian}_{\sigma=20}\}$$
$$\rho_{\text{range}} \in \{\text{l1, l2, Gaussian}_{\sigma=2}\}$$
were used to generate the various estimates.
- Briefly justify your decisions.

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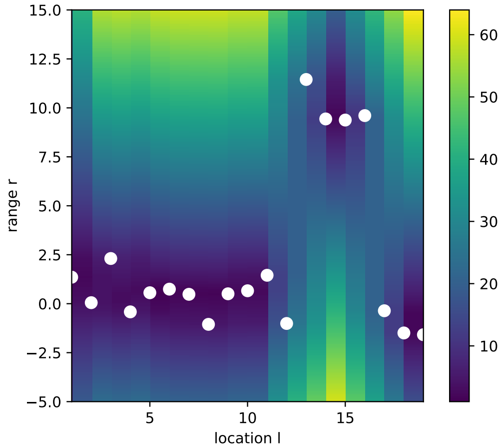
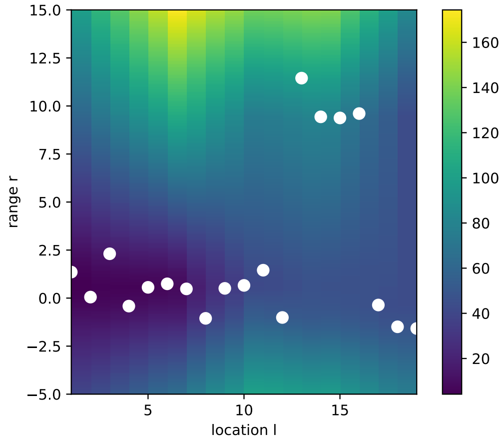
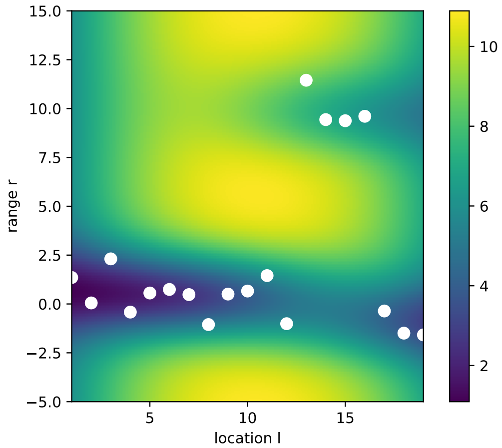


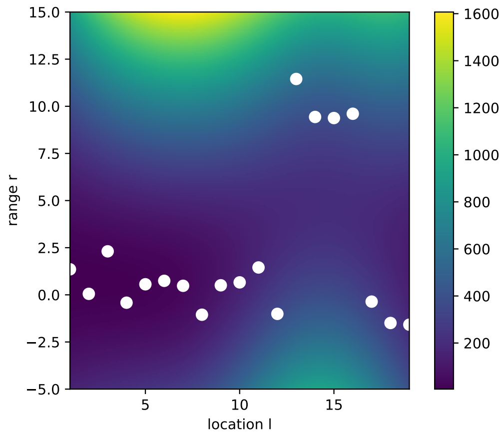
(a)



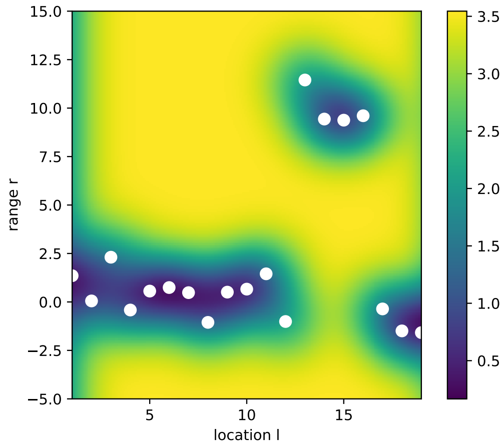
(b)



(c)



(d)



(e)

Abbildung 1

1.2 Linearity (4 points)

Consider the estimator, where we have made the dependency on the observation vector $o \in \mathbb{R}^n$ explicit:

$$\hat{\mu} : \mathbb{R} \times \mathbb{R}^n \mapsto \mathbb{R} :$$
$$\hat{\mu}(\tilde{l}, o) = \arg\min_r \sum_i w_{\text{spatial}}(\tilde{l} - l_i)\rho_{\text{range}}(r - o_i) \tag{5}$$

Task:

- Proof that there exists ρ^*_{range} such that $\hat{\mu}$ is linear in o for any choice of w_{spatial}.
  Hint: ρ^*_{range} is listed in Equation (4)

1.3 Implement Median Filters (4 points)

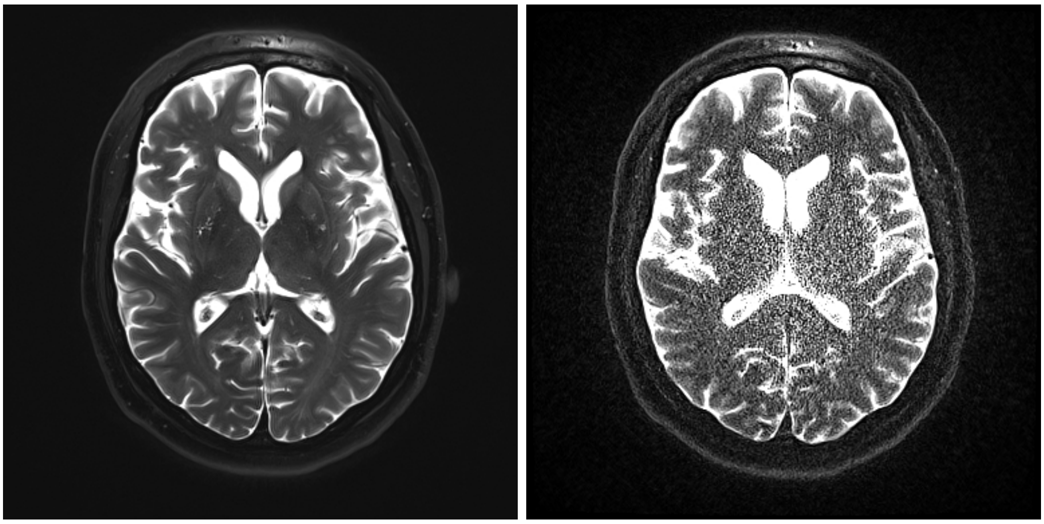


Abbildung 2: MRI brain scan without acceleration(left) and with an acceleration factor of 6 (right)

Abbildung 3: Image with artificial salt and pepper noise

When the acquisition time of magnetic resonance imaging (MRI) is speedup by a factor of R the signal to noise ratio (SNR) is reduced by a factor of $\frac{1}{\sqrt{R}}$. You can see these effects in Figure 2, where the right image was acquired R=6 times faster than the left image.

In this exercise we want to implement and apply a median filter to filter this noise. Please use the code provided on the website as a template for the following task.

Task:

- Implement the local_median function that loops over the image and replaces each pixel value with the median value of its neighboring pixels (within a box of length `kernel_size`). Do not use external libraries other than numpy. However, you may speedup your processing by compiling your code. You can find a simple numba jit tutorial at `http://numba.pydata.org/numba-doc/0.17.0/user/jit.html`.
- Compute the median filter of the fast MRI image with `kernel_size` = 3, 5, 7
- Compute the median filter of Figure 3 with `kernel_size` = 5, 11 and briefly discuss how the details and noise in the image are affected by the filter and `kernel_size` choice.
- Consider a white image patch with a horizontal black line of 1 pixel width. What is the result after filtering with a 3x3 median filter?
- Explain why the median filter is often said to be "edge-preserving".
- **Bonus (2 Points):** Develop an efficient vectorized implementation of the median filter.