Environmental Fluid Dynamics, Exercise Sheet 5

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To be returned until 17.12.2012, in the lecture, by mail to jana.schnieders@iwr.uni-heidelberg.de or to Speyerer Str. 6, 3rd floor (G 302 or H 306)

1 Tornado (30 Pts)

A tornado can be idealized as a Rankine vortex with a core of diameter 30 m. The tangential velocity of a Rankine vortex with circulation Γ and radius R is

$$u_\theta(r) = \begin{cases} \Gamma r/(2\pi R^2) & r \leq R, \\ \Gamma/(2\pi r) & r > R. \end{cases} \tag{1}$$

The remainder of the velocity components are identically zero, so that the total velocity field is $\mathbf{u} = u_\theta \, \mathbf{e}_\theta$. The gauge pressure at a radius of 15 m is -2000 N/m^2 (i.e., the absolute pressure is 2000 N/m^2 below atmospheric).

1.1

Show that the circulation around any circuit surrounding the core is 5485 m^2/s. [Hint: Apply the Bernoulli equation between infinity and the edge of the core.]

1.2

Such a tornado is moving at a linear speed of 25 m/s relative to the ground. Find the time required for the gauge pressure to drop from -500 to -2000 N/m^2. Neglect compressibility effects and assume an air temperature of 298 K. (Note that the tornado causes a sudden decrease of the local atmospheric pressure. The damage to structures is often caused by the resulting excess pressure on the inside of the walls, which can cause a house to explode.)

2 Kelvin's circulation theorem (30 Pts)

Show that

$$\frac{D}{Dt}\Gamma_a = 0 \tag{2}$$

with

$$\Gamma_a = \int (\omega + 2\mathbf{\Omega})\mathbf{dA}) \tag{3}$$

is an extension of Kelvin's circulation theorem in a rotating frame. Assume that $\mathbf{\Omega}$, the vorticity of the rotating frame, is constant, the flow is inviscid and barotropic and that the body forces are conservative. Explain the result physically.

3 Conservation of Potential Vorticity (30 Pts)

The equations for absolute vorticity in the ocean for frictionless flow are given by

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{4}$$

$$\frac{Dv}{Dt} - fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{5}$$

3.1

Show that the potential vorticity

$$\Pi = \frac{\zeta + f}{H} \tag{6}$$

is conserved along a fluid trajectory. ζ relative vorticity (of the eddy), f planetary vorticity and H water depth. (You will need the continuity equation)

3.2

An eddy in the North Atlantic has a vorticity of $\zeta = 10^{-5}$1/s. The water depth is 500m. The eddy is moving eastwards and the water depth is decreasing to 300m. What happens to the eddy?