Environmental Fluid Dynamics, Exercise Sheet 3

Christoph Garbe, Jana Schnieders

Wintersemester 2012/2013

Please submit by 26.11.2012, in the lecture, by mail to jana.schnieders@iwr.uni-heidelberg.de or to Speyerer Str. 6, 3rd floor (G 302 or H 306)

1 Reynolds number (30 Pts)

1.1 (10 Pts)

Derive the dimensionless group which is best suited to describing a flow as either laminar or turbulent. Please applying *Buckingham's Pi* theorem as introduced in the lecture. What is the common name for this group?

1.2 (10 Pts)

The Reynolds number describes the ratio of inertial and viscous forces. Explain this statement in more detail (use the Navier-Stokes-Equations in steady state). For which types of fluid mechanical processes is the Reynolds number important and why?

1.3 (10 Pts)

You want to experimentally examine the flow properties of the dolphin from exercise sheet 2. Since in-situ measurements of this type are challenging, you decide to built a very small model of the dolphin (your dolphin has a length of 2 cm). The flow properties should be comparable to the real world dolphin (2 m long, swimming with a velocity of 15 m/s in water). How would you design the experiment?

2 Ekman transport (30 Pts)

Ekman was the first to describe the influence of the Coriolis force on wind driven ocean currents. The simplified equations of movement including the Coriolis force are in this case:

$$\rho f u - \frac{\partial \tau_{yz}}{\partial z} = 0 \quad (1)$$

$$\rho f v + \frac{\partial \tau_{xz}}{\partial z} = 0 \quad (2)$$

Here $f = 2\Omega \sin\varphi$ is the Coriolis parameter und u and v are the velocities in x and y direction respectively. The Coriolis parameter is equal to twice the rotation rate Ω of the Earth ($\Omega = 7.2921 \times 10^{-5}$ rad/s) multiplied by the sine of the latitude φ. This leads to the so-called Ekman transport perpendicular to the direction of the flow. Especially in the gyres of the southern ocean upwelling regions arise due to so-called Ekman-Pumping.

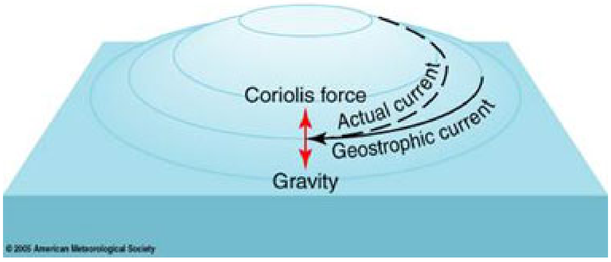

Figure 1: Ekman transport causes surface waters to move toward the central region of a subtropical gyre from all sides, producing a broad mound of water. Surface water begins flowing downhill. A balance develops between the Coriolis force and the force arising from the horizontal water pressure gradient such that surface currents flow parallel to the contours of elevation of sea level. This current is known as geostrophic flow. (©American Meteorological Society)

2.1 (15 Pts)

Show that the vertical velocity at the bottom of the wind-driven layer (z=-h) is given as

$$w_{-h} = \frac{1}{\rho f} \vec{k} \vec{\nabla} \times \vec{\tau} \quad (3)$$

where $\vec{\tau}$ is the wind stress, ρ the density, f the Coriolis parameter and $\vec{k}$ the vertical unit vector. (Hint: Start with the equation of continuity and use the given equations)

2.2 (15 Pts)

Calculate a typical vertical velocity related to the low pressure systems near the Antarctic Continent. Assume a wind velocity of $U = 10$ m/s. The wind stress may be calculated as $\tau = \rho_a C_d U$, where ρ_a is the density of air and $C_D = 1.410^{-3}$ is the non-dimensional drag coefficient.

3 Equations of motion (30 Pts)

The equations describing a fluid flow are:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + 2(\Omega \times \vec{u}) = -\frac{1}{\rho}\nabla p - \Omega \times (\Omega \times r) + \nu \nabla^2 \vec{u} - g\vec{k} \tag{4}$$

$$\frac{D}{Dt}\int_V \rho(e + \frac{1}{2}u_i^2)\mathrm{d}V = \int_V \rho g_i u_i \mathrm{d}V + \int_A \tau_{ij} u_i \mathrm{d}A_j - \int_A q_i \mathrm{d}A_i \tag{5}$$

$$\frac{\partial \rho}{\partial t} + \vec{u} \cdot \nabla \rho = -\rho \nabla \cdot \vec{u} \tag{6}$$

$$p = \rho R T \tag{7}$$

besides the variables used in the lecture, Ω is the rotation velocity.

1. Name each equation.

2. State the physical meaning of each term in the equations.

3. How is the enthalpy of the system related to these equations?

4. Which terms will be dropped due to the condition of stationarity?

5. Write down $\vec{u} \cdot \nabla$ and $\nabla \cdot \vec{u}$ in cartesian form and show that they are significantly different. Which of them is a scalar function and which is a differential operator.