Environmental Fluid Dynamics, Exercise Sheet 2

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To be returned on 15.11.2012, in the tutorials, by mail to jana.schnieders@iwr.uni-heidelberg.de or to Speyerer Str. 6, 3rd floor (G 302 or H 306)

1 Brunt-Väisälä frequency (40 Pts)

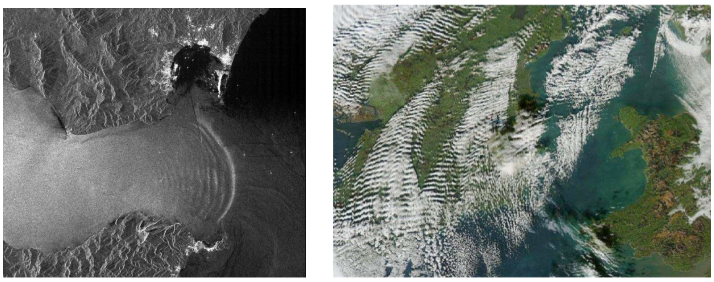


Figure 1: Left: Internal waves in the Strait of Gibralta. Right: Wave clouds over Ireland and the UK.

Internal waves appear frequently in statically stable fluids, e.g. under certain conditions in the ocean and the atmosphere.
Consider wind coming from the sea and blowing towards a mountain range. Assume that the surrounding air is stably stratified and in hydrostatic balance. Air parcels meeting the lee side of the mountain are now vertically displaced by a distance h'. At the new location the density can be described by $\rho = \rho_0 + \rho'$ and the pressure by $p = p_0 + p'$.

Due to the excess weight, the air parcel will tend to sink, due to inertia, to a location that is deeper than its initial location, where the ambient density is larger than that of the fluid parcel. Due to buoyancy, the particle will move upwards again, thus defining an oscillatory motion with a characteristic frequency, denoted Brunt-Väisälä frequency or simply buoyancy frequency.

1.1 20 Pts

Derive the expression for the Brunt-Väisälä frequency. (Hint: Assume an incompressible fluid with variable density for which the Boussinesq approximation applies. Start with the continuity equation in the correct form and Euler's equation. There are no density variations along the x- and y- axis only along the z- axis.)

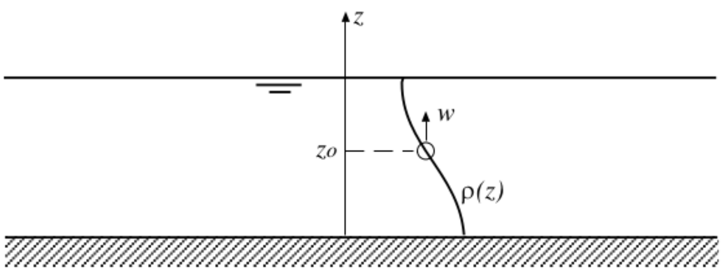


Figure 2: Fluid with stable stratification.

1.2 10 Pts

You are an oceanographer and have just measured the following density profile along the water coloum in the South Pacific (Antarctic Circumpolar Current):

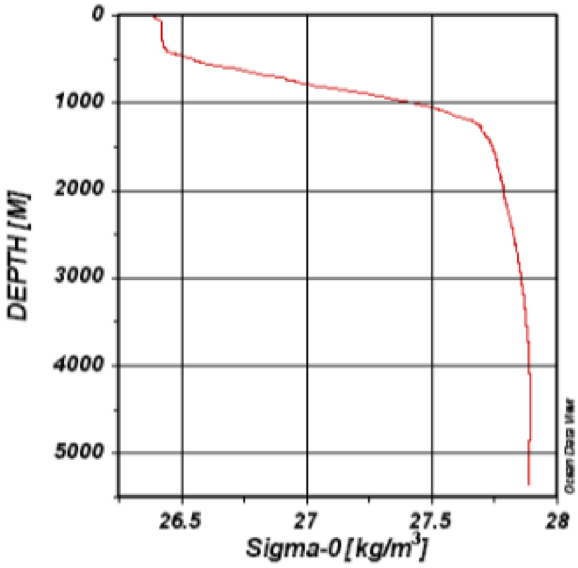


Figure 3: Density variation along the water coloum (sigma-0 denotes $\sigma_0 = \rho - 1000$ kg/m^3).

Sketch the corresponding profile of the Brunt-Väisälä frequency that you would expect to find here.

1.3 10 Pts

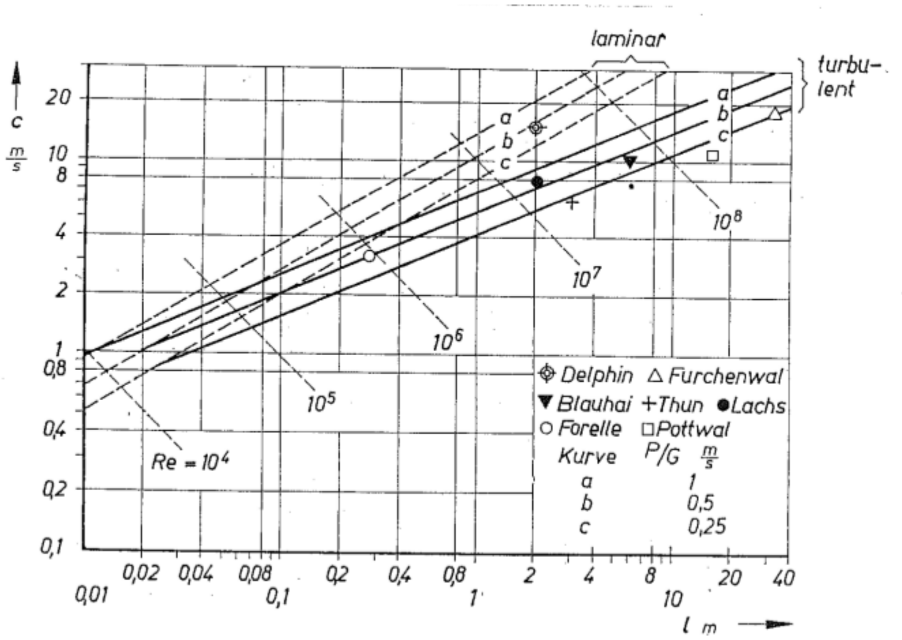
Estimate the density gradient from the profile above and calculate the maximum Brunt-Väisälä frequency in the profile.

2 Dolphins (20 Pts)

Dolphins can travel at surprisingly high speeds of up to 15 m/s. According to the 'Gray's Paradoxon' they would therefore need far higher muscle power than their muscles actually provide.

Calculate the highest possible swimming velocities in the cases of laminar and turbulent flow. (Hint: You need the equation for drag resistance. Assume an ellipsoid with a length of $l = 2$m and a width of $d = l/4$. Muscle power per kg can be assumed to be 0.5 W/kg. The drag coefficients

are:
$c_f = 0.074/Re^{1/5}$ for turbulent flow, and
$c_f = 1.328/Re^{1/2}$ for laminar flow.
What does this mean for the dolphin?



3 Divergence and Vorticity (20 Pts)

The Stream function of a flow is defined by:

$$\frac{\partial \psi}{\partial y} = -u; \frac{\partial \psi}{\partial x} = v \tag{1}$$

The velocity potential is defined by:

$$\nabla \phi = \vec{u} \tag{2}$$

3.1

Show that:
Every flow that is described by a stream function has zero divergence.
Every flow that is described by a potential has zero vorticity.

3.2

Express:
The vorticity in terms of the stream function.
The divergence in terms of the potential.

3.3

Draw the velocity vectors and streamlines and calculate vorticity and divergence of the following velocity fields:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}; \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} cosy \\ sinx \end{pmatrix} \tag{3}$$