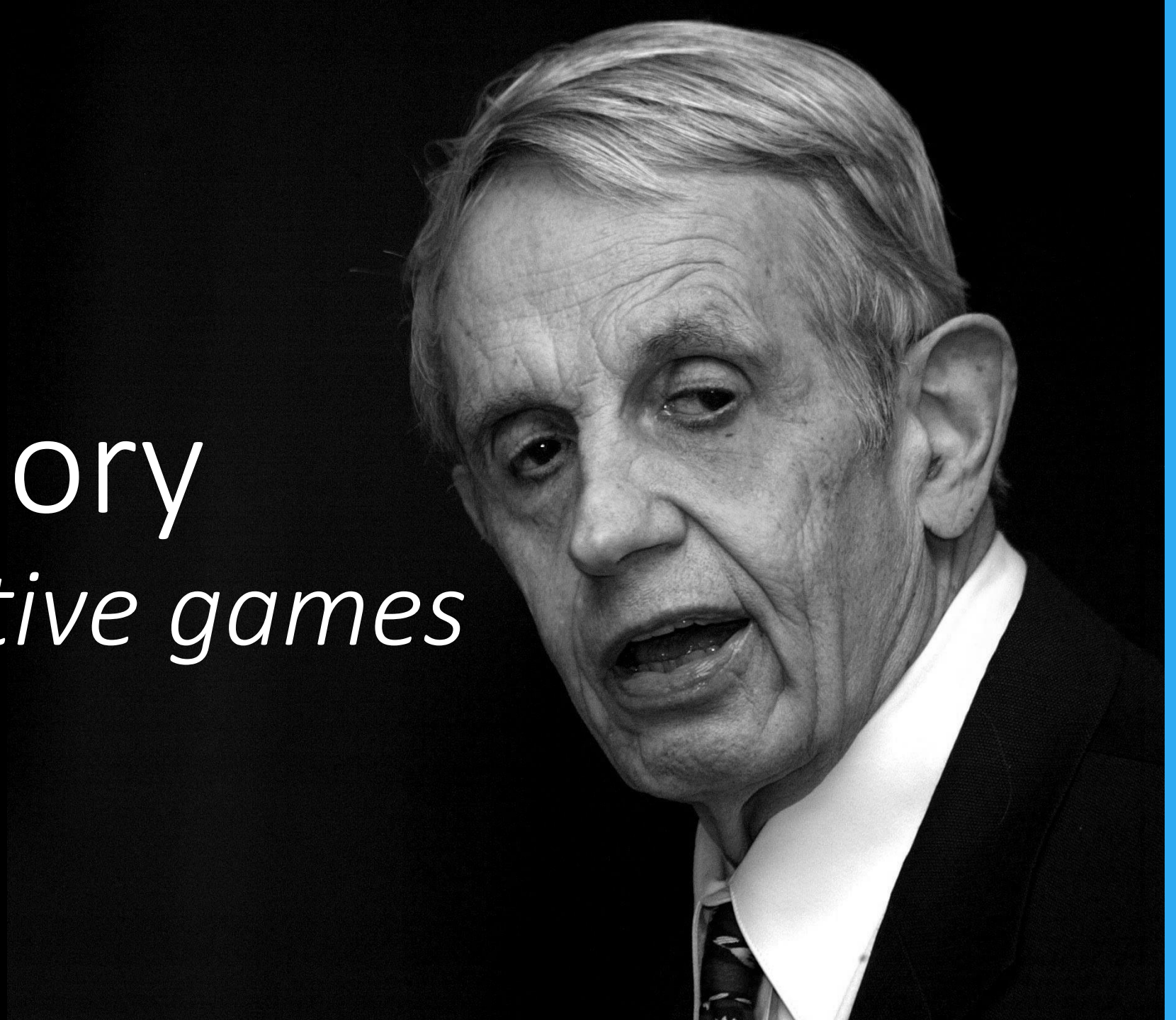
Game Theory

Non-cooperative games



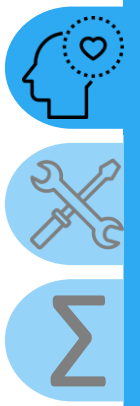
Speaker:

Philipp de Sombre

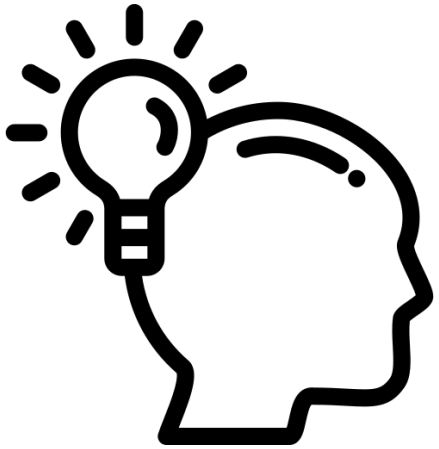
Outline

- Why should I care about Game Theory?
- How can I use Game Theory?
- What did I learn?

Why should I care about Game Theory?



Motivation



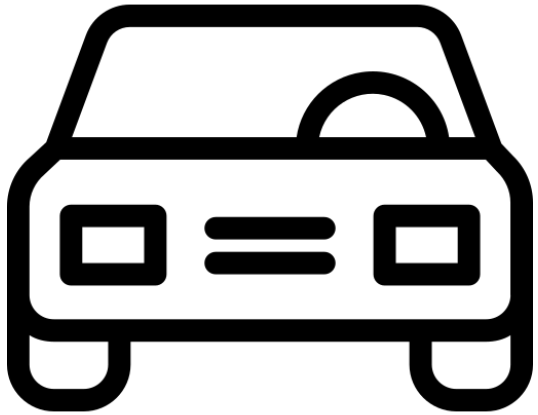
Mathematically appealing:

- Exciting thought processes
- Interesting solution concepts
- Different perspective on problems

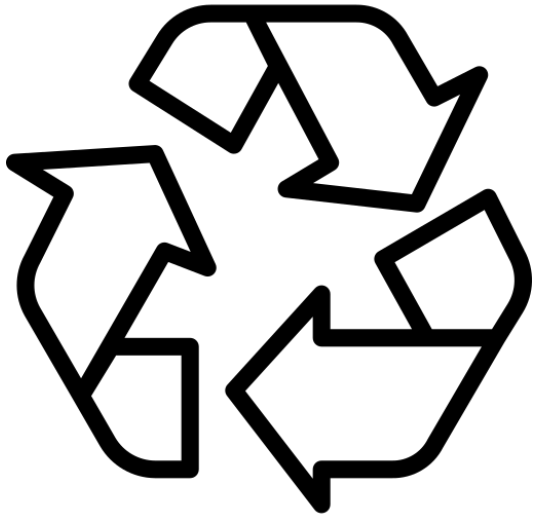
Real-world implications:

- Problems can be modeled as games
- Used in a lot of fields

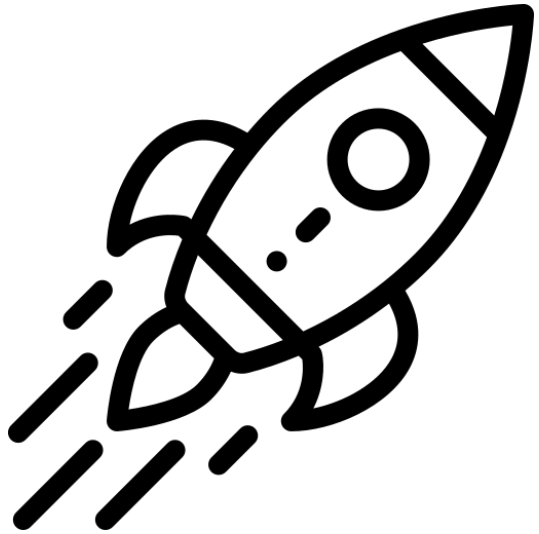
Real-world examples



Buying a car [3]
-> Personal application



Environment [4]
-> Public goods problem



Arms race [5]
-> Dilemma situation

Undertanding is the first step of solving.

How can I use Game Theory?

Fundamentals – Research fields

Normative

What would be the best thing to do?

- Rational Agent
- Mathematics, Economy

Descriptive

What do people actually do?

- Real People
- Psychology, Social sciences

Fundamentals – A Game

- *A strategic game is a model of **interacting decision-makers**. […] The model **captures interaction** between the players by allowing each player to be affected by the actions of all players, not only her own action. Each player has **preferences** about the action profile […].* ([1] p. 11)

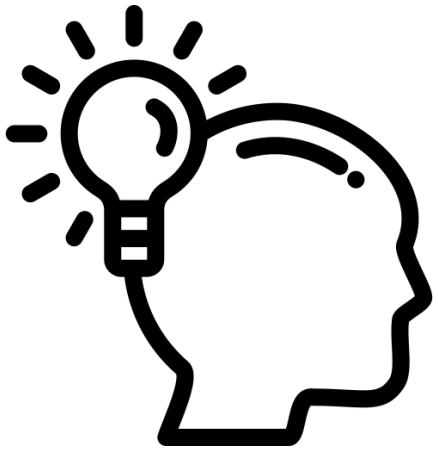
$$\boldsymbol{Game = \{Players, Strategies, Utilities\}}$$

- ***What examples can you think of?***

Normal form games - Assumptions

Static:

- Actions are “simultaneous”



Complete Information:

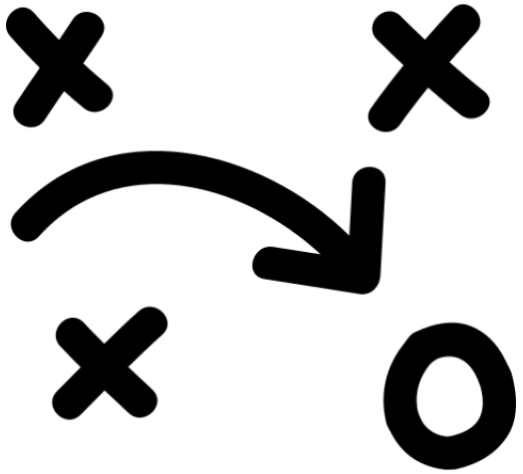
- All players know “everything”

Normal form games - elements

$$\boldsymbol{G = \{P, S, U\}}$$

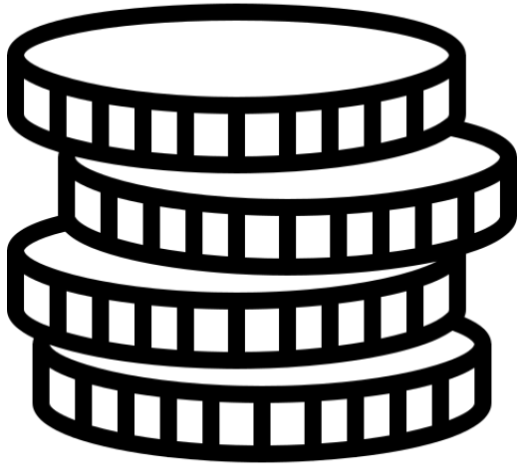
Players:

$P = \{1,2, \dots, n\}$



Pure Strategies:

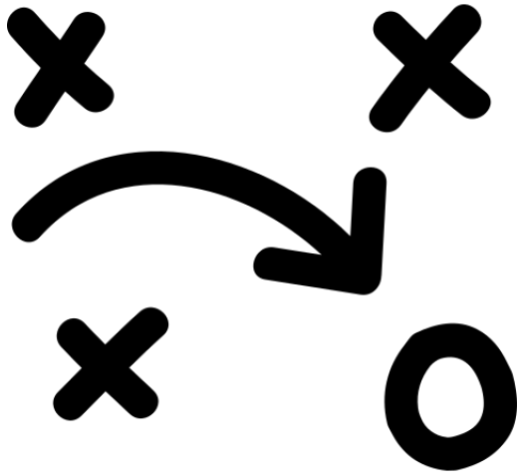
$S = \{S_1, S_2, \dots, S_n\}$

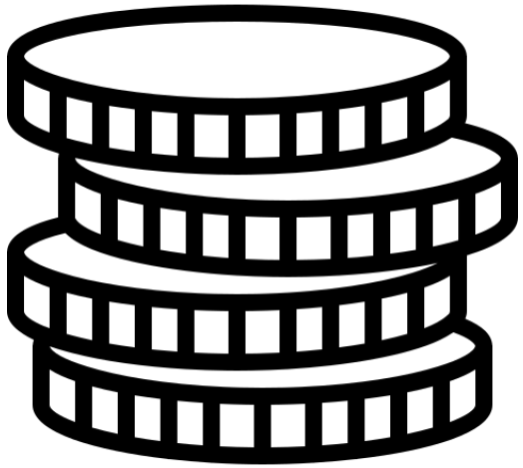


Payoff functions:

$U = \{u_1, u_2, \dots, u_n\}$

Normal form games - elements





Pure Strategies:

$$S = \{S_1, S_2, \dots, S_n\}$$
$$S_i = \{1,2, \dots, k\}$$
$$\vec{s} = (s_1, s_2, \dots, s_n)$$
$$s_i \in S_i$$

Payoff functions:

$$U = \{u_1, u_2, \dots, u_n\}$$
$$u_i: S_1 \times S_2 \times \cdots \times S_n \to \mathbb{R}$$

Normal form games - payoff matrix

		Player B	
		Even	Odd
Player A	Even	1,0	0,1
	Odd	0,1	1,0

Players | Pure Strategies | Payoffs

A wins when sum is even

Games

- https://aifg.desomb.re

Game 0: Even-Odd

		Player B: Even	Player B: Odd
Player A	Even	1,0	0,1
	Odd	0,1	1,0

What did you choose and why?

Game 1: Rock, Paper, Scissor

		Player B		
		Rock	Paper	Scissor
Player A	Rock	1,1	0,3	3,0
	Paper	3,0	1,1	0,3
	Scissor	0,3	3,0	1,1

What did you choose and why?

Game 2: Shared fridge

		Player B	
		Low Quality	High Quality
Player A	Low Quality	1,1	2,0
	High Quality	0,2	3,3

What did you choose and why?

Best response

Idea:

What is the best thing to do for a given strategy of the opponent?

Definition:

The strategy $s_i \in S_i$ is player i's best response to his opponents' strategies $s_{-i} \in S_{-i}$ if:

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \quad \forall s_i' \in S_i$$

([2], p. 70)

Best response

$$u_i(s_i, s\_i) \geq u_i(s_i', s\_i) \forall s_i' \in S_i$$

Player A \ Player B	Low Quality	High Quality
Low Quality	1,1	2,0
High Quality	0,2	3,3

Game 3: Prisoner's dilemma

		Player B	
		Say nothing	talk
Player A	Say nothing	-1,-1	-3,0
	talk	0,-3	-2,-2

What did you choose and why?

Dominated strategies

Idea:

Is there a strategy thats always better than the others?

Definition:

*Let $s_i \in S_i$ and $s'_i \in S_i$ be possible strategies for player **i**. We say that s'_i is dominated by s_i if for any possible combination of the other players' strategies, $s_{-i} \in S_{-i}$, player i's payoff from s'_i is less than that from s_i.*

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}) \quad \forall s_{-i} \in S_{-i}$$

≥: Weakly dominated

>: Strictly dominated

([2], p. 60 ff)

Dominated strategies

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \ \forall s_{-i} \in S_{-i}$$

		Player B: Say nothing	Player B: talk
Player A	Say nothing	-1,-1	-3,0
	talk	0,-3	-2,-2

is dominated ⇒ a rational Player will never play it

Game 4: Diner's dilemma

		Player B	
		Expensive Meal	Cheap Meal
Player A	Expensive Meal	2,2	3,0
	Cheap Meal	0,3	2,2

What did you choose and why?

The Nash equilibrium

- Solution of a non-cooperative game
- Stable strategy profile

  -> No motivation for deviation for players
- may appear non-rational

  e.g.: Prisoner's dilemma
- Proof of existence for finite games by John Nash (1950)

The Nash equilibrium

Definition:

A Nash equilibrium is a strategy profile $\vec{s}^{\,*}$ *with the property that no player* $\boldsymbol{i}$ *can do better by choosing an action different from* $\boldsymbol{s_i}^{*}$*, given that every other player* $-\boldsymbol{i}$ *adheres to* $\boldsymbol{s_{-i}}^{*}$*.*

$$u_i(\vec{s}^{\,*}) \geq u_i(s_i, s^*_{-i}) \forall\, s_i \in S_i, \forall\, i \in P$$

([1], p. 20 ff)

Finding the Nash equilibrium

$$u_i(\vec{s}^{\,*}) \geq u_i(s_i, s^*_{-i}) \forall\, s_i \in S_i, \forall\, i \in P$$

		Player B: Expensive Meal	Player B: Cheap Meal
Player A	Expensive Meal	2,2	3,0
	Cheap Meal	0,3	2,2

Game 5: Stag hunt

		Player B: Stag	Player B: Hare
Player A	Stag	3,3	0,2
	Hare	2,0	1,1

⇒2 Nash eq.

What did you choose and why?

Game 6: Partner work

		Player B: Work hard	Player B: Goof off
Player A	Word hard	2,2	0,3
	Goof off	3,0	1,1

What did you choose and why?

Nash eq.: but not the „best" Solution for all Players

Game 7: Marble game

		Player B		
		Red	Blue	Yellow
Player A	Red	2,2	0,3	0,1
	Blue	3,0	2,2	2,0
	Yellow	1,0	0,2	1,1

What did you choose and why?

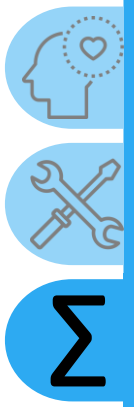
Game 8: Battle of sexes

		Player B	
		Football	Rom-com
Player A	Football	1,2	0,0
	Rom-com	0,0	2,1

? Nash eq. in pure Strats ? -> mixed Strats.

What did you choose and why?

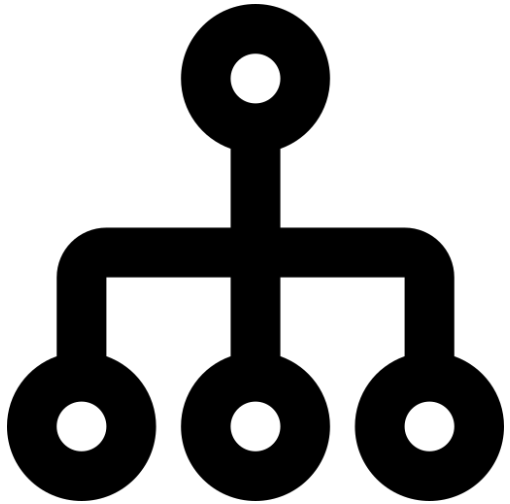
What did I learn?



Further topics

$$\frac{2}{3}$$

Mixed strategies
->Probability



Sequential games
->Not simultaneous

Incomplete/Imperfect information
->Uncertainty

Cooperative games
->Contracts
->Solution for dilemmas

Summary

Overview:

- Research fields
- Definition of a Game

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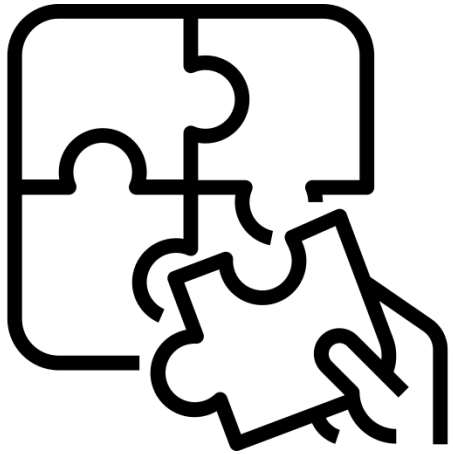
Elements:

- Players
- Pure strategies
- Payoffs



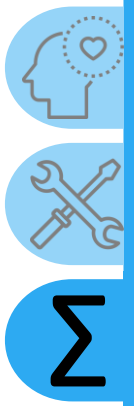
Solution concepts:

- Best response
- Dominated Strategies
- Nash equilibrium

Questions

Results of the Games

https://aifg.desomb.re/#/scoreboard



Sources - Literature

- [1] Osborne, Martin J. *An introduction to game theory*. New York: Oxford University Press, 2004. Print.
- [2] Tadelis, Steve. *Game theory : an introduction*. Princeton Oxford: Princeton University Press, 2013. Print.
- [3] Duronio , Ben. *How To Use Game Theory To Get A Great Deal On A Car.* Business Insider, 03.04.2012. https://www.businessinsider.com/game-theory-buy-car-2012-4, accessed on 02.05.2019
- [4] Ray, Indrajit & Yo Dd, York. (2001*). Game Theory and the Environment: Old Models, New Solution Concepts*.
- [5] Güth, Werner. *Game theory and the nuclear arms race - The strategic position of Western Europe* - , European Journal of Political Economy 4 (1988), 245-261.

Sources - Images

- John Nash: https://timedotcom.files.wordpress.com/2015/05/john-nash-2.jpg?quality=85

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