Efficient Energy Maximization Using Smoothing Technique

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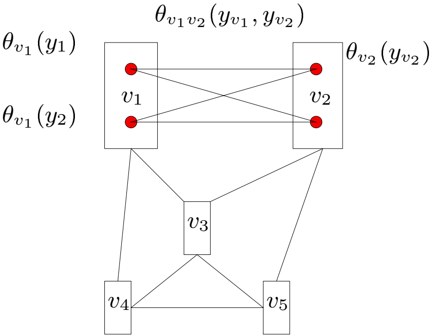
Energy Maximization Problem

$G = (\mathcal{V}, \mathcal{E}),\ v \in \mathcal{V},\ vv' \in \mathcal{E}$
$y_v \in \mathcal{Y}$ - *labels*, $y = (y_v, v \in \mathcal{V}) \in \mathcal{Y}^{\mathcal{V}} \equiv \mathcal{Y}^{|\mathcal{V}|}$
$\theta_t(y_v)$ - *unary potentials*, $\theta_{vv'}(y_v, y_{v'})$*-binary potentials*

$$y^* = \arg\max_{y \in \mathcal{Y}^{\mathcal{V}}} \left[\sum_{v \in \mathcal{V}} \theta_v(y_v) + \sum_{vv' \in \mathcal{E}} \theta_{vv'}(y_v, y_{v'}) \right] = \arg\max_{y \in \mathcal{Y}^{\mathcal{V}}} E(\theta, y)$$

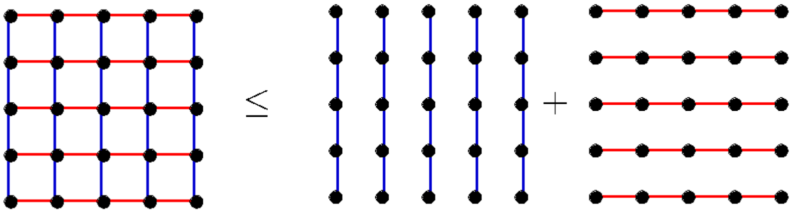


Dual Decomposition Approach to the Energy Maximization Problem

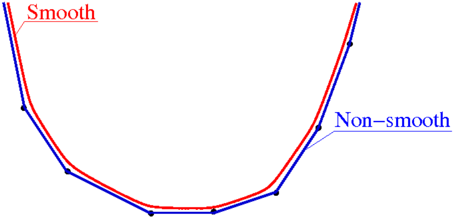
$$\theta = \theta^1 + \theta^2 \quad \Leftrightarrow \quad E(\theta, y) = E_1(\theta^1, y) + E_2(\theta^2, y)$$

$$\max_{y \in \mathcal{Y}^{\mathcal{V}}} E(\theta, y) \leqslant \max_{y \in \mathcal{Y}^{\mathcal{V}}} E_1(\theta^1, y) + \max_{y \in \mathcal{Y}^{\mathcal{V}}} E_2(\theta^2, y)$$

$$\max_{y \in \mathcal{Y}^{\mathcal{V}}} E(\theta, y) \leqslant \min_{\theta^1 + \theta^2 = \theta} \left[\max_{y \in \mathcal{Y}^{\mathcal{V}}} E_1(\theta^1, y) + \max_{y \in \mathcal{Y}^{\mathcal{V}}} E_2(\theta^2, y) \right]$$



Solving the Problem Efficiently

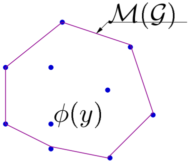


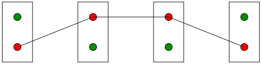
For our problem an effective and uniformly tight smoothing is possible.

Optimization method	**Convergence**
Non-smooth optimization (sub-gradient)	$O(1/\varepsilon^2)$
A smooth gradient descent	$O(\frac{L}{\varepsilon})$
↳ Applied to the non-smooth function	$L = \frac{1}{\varepsilon} \Rightarrow O(\frac{1}{\varepsilon^2})$
An optimal smooth first-order optimization:	$O(\sqrt{\frac{L}{\varepsilon}})$
↳ Applied to the non-smooth function	$L = \frac{1}{\varepsilon} \Rightarrow O(\frac{1}{\varepsilon})$

Outline

Energy Maximization Problem





$$\phi(y) = \begin{pmatrix} 01 & & 10 & & 10 & & 01 \\ & 0010 & & 1000 & & 0100 & \end{pmatrix}$$

$$\max_{y \in \mathcal{Y}^{\mathcal{V}}} \left[\sum_{v \in \mathcal{V}} \theta_v(y_v) + \sum_{vv' \in \mathcal{E}} \theta_{vv'}(y_v, y_{v'}) \right] \Rightarrow \max_{y \in \mathcal{Y}^{\mathcal{V}}} \sum_{c \in \mathcal{C}} \theta_c(y_c)$$

$$= \max_{y \in \mathcal{Y}^{\mathcal{V}}} \sum_{c \in \mathcal{C}} \langle \theta_c, \phi_c(y) \rangle = \max_{y \in \mathcal{Y}^{\mathcal{V}}} \langle \theta, \phi(y) \rangle = \max_{\mu \in \mathcal{M}(\mathcal{G})} \langle \theta, \mu \rangle$$

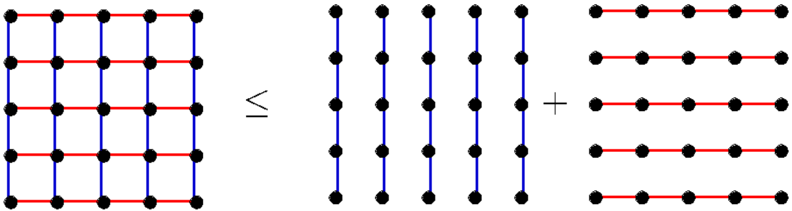
$$\phi_{(c,x')}(y) = \begin{cases} 1, & \text{if } y_c = x' \\ 0, & \text{otherwise} \end{cases}, \quad \mathcal{M}(\mathcal{G}) = \text{conv}\{\phi(y) : y \in \mathcal{Y}^{\mathcal{V}}\}$$

Decomposition

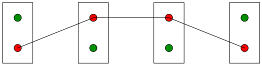
$$\theta = \theta^1 + \theta^2 \quad \Leftrightarrow \quad E(\theta, y) = E_1(\theta^1, y) + E_2(\theta^2, y)$$

$$\theta_c^1(\vartheta_c) = \begin{cases} \frac{\theta_c}{2} + \vartheta_c, & c \in \mathcal{V} \\ \theta_c, & c \in \mathcal{E}^1 \\ 0, & \text{otherwise} \end{cases} \qquad \theta_c^2(\vartheta_c) = \begin{cases} \frac{\theta_c}{2} - \vartheta_c, & c \in \mathcal{V} \\ \theta_c, & c \in \mathcal{E}^2 \\ 0, & \text{otherwise} \end{cases}$$

$$\max_{\mu \in \mathcal{M}(\mathcal{G})} \langle \theta, \mu \rangle \leqslant \max_{\mu^1 \in \mathcal{M}(\mathcal{G}^1)} \left\langle \theta^1(\vartheta), \mu^1 \right\rangle + \max_{\mu^2 \in \mathcal{M}(\mathcal{G}^2)} \left\langle \theta^2(\vartheta), \mu^2 \right\rangle$$



Smoothing



$$\phi(y) = (01 \quad 10 \quad 10 \quad 01)$$
$$0010 \quad 1000 \quad 0100$$

$$\max\{a_1, \ldots, a_n\} \simeq \rho \log\{e^{a_1/\rho} + \cdots + e^{a_n/\rho}\}$$

$$\max\{a_1, \ldots, a_n\} \leqslant \rho \log\{e^{a_1/\rho} + \cdots + e^{a_n/\rho}\} \leqslant \max\{a_1, \ldots, a_n\} + \rho \log n$$

$$U_{\mathcal{G}^i}(\vartheta) = \max_{\mu \in \mathcal{M}(\mathcal{G}^i)} \langle \theta^i(\vartheta), \mu \rangle = \max_{y \in \mathcal{Y}^{\mathcal{V}}} \langle \theta^i(\vartheta), \phi(y) \rangle$$

$$\hat{U}_{\mathcal{G}^i}(\vartheta, \rho) = \rho \log \sum_{y \in \mathcal{Y}^{\mathcal{V}}} \exp(\langle \theta^i(\vartheta)/\rho, \phi(y) \rangle)$$

$$U_{\mathcal{G}^i}(\vartheta) \leqslant \hat{U}_{\mathcal{G}^i}(\vartheta, \rho) \leqslant U_{\mathcal{G}^i}(\vartheta) + \rho \log |\mathcal{Y}^{\mathcal{V}}| = U_{\mathcal{G}^i}(\vartheta) + \rho |\mathcal{V}| \log |\mathcal{Y}|$$

$$\left(\frac{\partial \hat{U}_{\mathcal{G}^i}(\vartheta)}{\partial \vartheta_v(y_v)} \right) = \pm \rho \left(\frac{\sum_{y \in \mathcal{Y}(c, y_v)} \exp(\langle \theta^i(\vartheta)/\rho, \phi(y) \rangle)}{\hat{U}_{\mathcal{G}^i}(\theta^i(\vartheta), \rho))} \right)$$

Smoothing

$$U(\vartheta) = U_{\mathcal{G}^1}(\vartheta) + U_{\mathcal{G}^2}(\vartheta)\,.$$

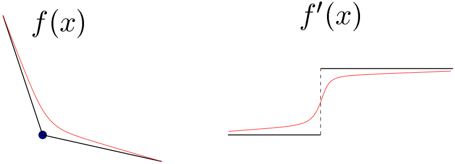
$$\hat{U}(\vartheta, \rho) = \hat{U}_{\mathcal{G}^1}(\vartheta, \rho) + \hat{U}_{\mathcal{G}^2}(\vartheta, \rho)$$

$$U(\vartheta) \leqslant \hat{U}(\vartheta, \rho) \leqslant U(\vartheta) + 2\rho|\mathcal{V}| \log|\mathcal{Y}|$$

$$\frac{\partial \hat{U}(\vartheta, \rho)}{\partial \vartheta} = \frac{\partial \hat{U}_{\mathcal{G}^1}(\vartheta, \rho)}{\partial \vartheta} + \frac{\partial \hat{U}_{\mathcal{G}^2}(\vartheta, \rho)}{\partial \vartheta}$$

Optimization: Lipschitz-Continuous Gradient

$f : \mathbb{R}^n \to \mathbb{R}$ – differentiable

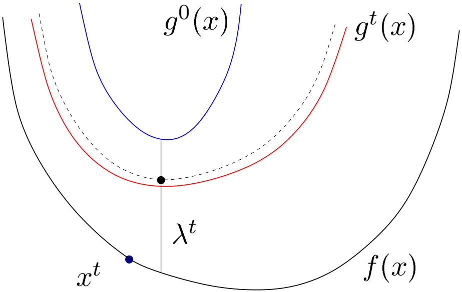


$$||\nabla f(x) - \nabla f(z)||_* \leqslant L||x - z||$$

Lemma (special case of Nesterov04)

Function $\hat{U}(\vartheta)$ is convex and Lipschitz-continuous with $L \leqslant \frac{2}{\rho}|\mathcal{V}|$

Optimization: Estimate Sequence [Nesterov83]



$$g^t(x) \leqslant (1-\lambda^t)f(x) + \lambda^t g^0(x), \quad \lambda^t \to 0$$
$$\{x^t\}: \quad f(x^t) \leqslant g^{t*} \equiv \min_{x\in\mathbb{R}^n} g^t(x)$$
$$f(x^t) - f^* \leqslant \lambda^t (g^0(x^*) - f^*) \to 0$$

Optimization: Algorithm

Algorithm (a variant of the algorithm 2.2.6 from Nesterov83)

$\gamma^t, \alpha^t, \omega^t, \in \mathbb{R}$; $x^t, u^t, z^t \in \mathbb{R}^n$, t – *an iteration counter.*

- *Choose* $x^0 \in \mathbb{R}^n$ *and* $\omega^0, \gamma^0 > 0$, $z^0 = u^0$.
- t*-th iteration* ($t \geqslant 0$).
  1. *Compute* $f(z^t)$ *and* $\nabla f(z^t)$.
  2. *Find possibly small* ω^t *such that*

$$f(x^t) \leqslant f(z^t) - \frac{1}{2\omega^t}||\nabla f(z^t)||^2 \text{ ,where } x^t = z^t - \frac{1}{\omega^t}\nabla f(z^t)\,.$$

  3. *Compute* $\alpha^t \in (0,1)$ *from the quadratic equation* $\omega^t(\alpha^t)^2 = (1-\alpha^t)\gamma^t$. *Set* $\gamma^{t+1} = (1-\alpha^t)\gamma^t$.
  4. *Set* $u^{t+1} = \frac{(1-\alpha^t)\gamma^t u^t - \alpha^t \nabla f(z^t)}{\gamma^{t+1}}$.
  5. *Choose* $z^{t+1} = \frac{\alpha^t \gamma^t u^{t+1} + \gamma^{t+1} x^t}{\gamma^t}$

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Optimization: Algorithm properties

Lemma (for any convex function with a Lipschitz-continuous gradient)

Condition at the step 2 in the algorithm is fulfilled for any $\omega^t \geqslant L$.

Lemma (modification of Nesterov83)

Convergence speed of the algorithm is $O(\frac{\sqrt{\omega^{*t}}}{t^2})$, *where* $\omega^{*t} = \max_{k \leqslant t} \omega^k$.

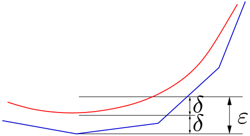
Corollary (modification of Nesterov83)

$\varepsilon = O(\frac{\sqrt{\omega^{*t}}}{t^2}) \Rightarrow t = O(\sqrt{\frac{\omega^{*t}}{\varepsilon}}); \quad \omega^{*t} \leqslant L = O(\frac{1}{\rho})$

Theorem (Algorithm convergence rate)

$$t = O\left(\sqrt{\frac{1}{\rho\varepsilon}}\right), \text{ if } \rho \sim \varepsilon \Rightarrow t = O\left(\frac{1}{\varepsilon}\right)$$

Smoothing selection



$$U(\vartheta) \leqslant \hat{U}(\vartheta, \rho) \leqslant U(\vartheta) + 2\rho|\mathcal{V}| \log|\mathcal{Y}|$$
$$\delta(\rho) = 2\rho|\mathcal{V}| \log|\mathcal{Y}|$$

[Nesterov04](the worst-case estimation) : $\rho = \frac{\varepsilon}{4|\mathcal{V}| \log|\mathcal{Y}|} \Rightarrow \delta(\rho) \leqslant \frac{\varepsilon}{2}$.

We estimate $\delta'(\rho) = \hat{U}(\vartheta, \rho) - U(\vartheta)$ for ϑ and use ρ such that $\delta'(\rho) \leqslant \frac{\varepsilon}{2}$.

Optimization: L Estimation

How large is $L = \frac{2}{\rho}|\mathcal{V}|$ for a typical setting?

$$\rho = 1,\ |\mathcal{V}| = 10 \times 10,\ \Rightarrow L = 2 \cdot 10^2$$

$$\rho = 1,\ |\mathcal{V}| = 100 \times 100,\ \Rightarrow L = 2 \cdot 10^4$$

Typical setting:

$$\rho = 1,\ |\mathcal{V}| = 400 \times 700,\ \Rightarrow L \approx 2 \cdot 10^5$$

We dynamically estimate L. Typical values are $1 - 1000$.

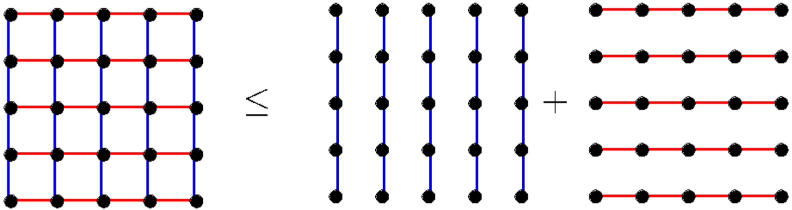
Optimization: ω^t Estimation

Algorithm (typical scheme of a linear search, implemented also in Nesterov05)

Input ω^t, $\nabla f(z^t)$, *parameters* $a > 1$ *and* $b > 1$, *output* ω^{t+1}, x^{t+1}.

1. *Set* $\omega^* = \omega^t / b$
2. *Calculate* $x^* = z^t - \frac{1}{\omega^*} \nabla f(z^t)$.
3. *If* $f(x^*) \leqslant f(z^t) - \frac{1}{2\omega^*} ||\nabla f(z^t)||^2$ **End.**
   Else assign $\omega^* = a \cdot \omega^*$ *goto step 2.*

Lower Bound: Standard Approach (covers only an LP-tight Case)



$\max\{E(\theta, y^1), E(\theta, y^2)\}$ $\leq$ y^1 $+$ y^2

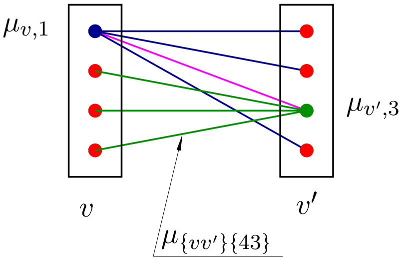
Lower Bound: Known Facts

$$\max_{\mu \in \mathcal{M}(\mathcal{G})} \langle \theta, \mu \rangle \leqslant \max_{\mu^1 \in \mathcal{M}(\mathcal{G}^1)} \left\langle \theta^1(\vartheta), \mu^1 \right\rangle + \max_{\mu^2 \in \mathcal{M}(\mathcal{G}^2)} \left\langle \theta^2(\vartheta), \mu^2 \right\rangle$$

if $\mathcal{G}^1$, $\mathcal{G}^2$ – trees, then:

$$\max_{\mu \in \mathcal{L}(\mathcal{G})} \langle \theta, \mu \rangle = \max_{\mu^1 \in \mathcal{M}(\mathcal{G}^1)} \left\langle \theta^1(\vartheta), \mu^1 \right\rangle + \max_{\mu^2 \in \mathcal{M}(\mathcal{G}^2)} \left\langle \theta^2(\vartheta), \mu^2 \right\rangle$$

where $\mathcal{L}(\mathcal{G})$ - a local polytope. Moreover, $\mathcal{M}(\mathcal{G}^i) = \mathcal{L}(\mathcal{G}^i)$.

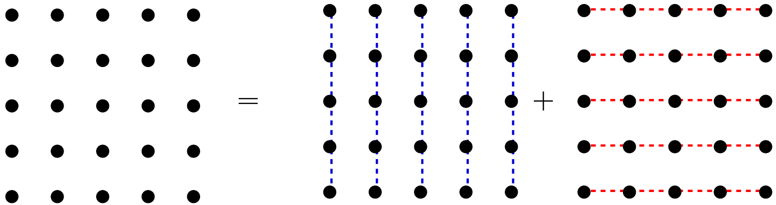


$$\begin{cases} \sum_{y' \in \mathcal{Y}} \mu_{\{vv'\},\{yy'\}} = \mu_{v,y} \\ \sum_{y \in \mathcal{Y}} \mu_{v,y} = 1 \\ \mu \geqslant 0 \end{cases}$$

Lower Bound: Our Approach (covers all cases)

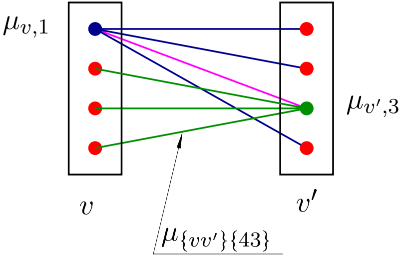
$$\mu_v^i = \pm\left(\frac{\partial \hat{U}_{\mathcal{G}^i}(\vartheta)}{\partial \vartheta_v(y_v)}\right) = \rho\left(\frac{\sum_{y\in\mathcal{Y}(c,y_v)} \exp(\langle \theta^i(\vartheta)/\rho, \phi(y)\rangle)}{\hat{U}_{\mathcal{G}^i}(\theta^i(\vartheta),\rho))}\right)$$

$\nabla\hat{U} = \mu_v^1 - \mu_v^2 \to 0$ close to the optima



$$\mu_v = \frac{\mu_v^1 + \mu_v^2}{2}$$

Lower Bound: Recovering $\mu_{vv'}$



$$\max \langle \theta, \mu \rangle$$

$$\begin{cases} \sum_{y' \in \mathcal{Y}} \mu_{\{vv'\},\{yy'\}} = \mu_{v,y} \\ \sum_{y \in \mathcal{Y}} \mu_{v,y} = 1 \\ \mu \geqslant 0 \end{cases}$$

hm... $\Downarrow$

$$\begin{cases} \sum_{y' \in \mathcal{Y}} \mu_{\{vv'\},\{yy'\}} = \mu_{v,y} \\ \text{for fixed } \mu_{v,y} = \frac{\mu^1_{v,y} + \mu^2_{v,y}}{2} \end{cases}$$

LP - Transportation problem!

Lower Bound: Theorem

Theorem

Let $\mu^{1,t} \in \mathcal{L}(\mathcal{G})$ *and* $\mu^{2,t} \in \mathcal{L}(\mathcal{G})$, $t = 1, \ldots \infty$ *be two sequences meeting the following conditions:*

1. $\mu_v^{1,t} - \mu_v^{2,t} \to 0,\ v \in \mathcal{V}$
2. $\left\langle \theta^i(\vartheta), \mu^{i,t}\big|_{\mathcal{L}(\mathcal{G}^i)} \right\rangle - \max_{\mu \in \mathcal{L}(\mathcal{G}^i)} \left\langle \theta^i(\vartheta), \mu \right\rangle \to 0,\ i = 1, 2$
3. $$\left\langle \theta, \frac{\mu^{1,t} + \mu^{2,t}}{2} \right\rangle = \max_{\mu \in \mathcal{L}(\mathcal{G})} \langle \theta, \mu \rangle \tag{1}$$
   $$\text{s.t. } \mu_v = \frac{\mu_v^{1,t} + \mu_v^{2,t}}{2},\ v \in \mathcal{V}$$

Then

$$0 \leqslant U^* - \left\langle \theta, \frac{\mu^{1,t} + \mu^{2,t}}{2} \right\rangle \to 0 \tag{2}$$

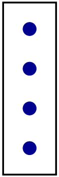
Implementation: Numerical Issues

$\theta_{c,1}$

$\theta_{c,2}$

$\vdots$

$\theta_{c,n}$



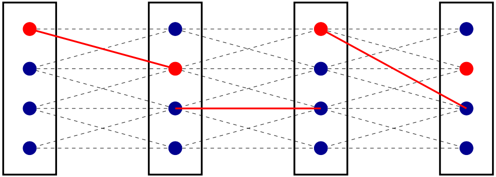
$\rightarrow$?

$\exp(\theta_{c,1})$

$\vdots$

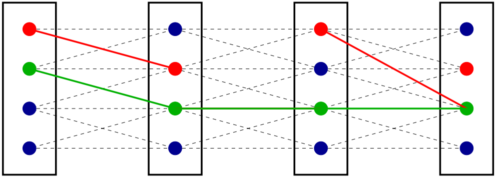
$\exp(\theta_{c,n})$

$$\theta^*_{c,i} = \theta_{c,i} - \max_{j=\overline{1,n}} \theta_{c,j}$$



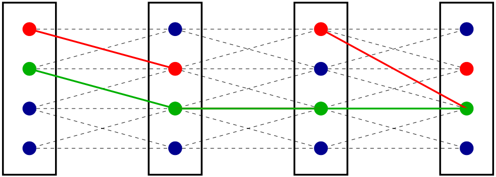
$$\exp\langle \theta^*/\rho, \phi(y)\rangle \xrightarrow[\rho\to 0]{} 0 \quad \forall y \in \mathcal{Y}^{\mathcal{V}}$$

Implementation: Numerical Issues



$$\theta^*_{c,y} = \theta_{c,y} - \theta_{c,y^*}$$
$$y^* = \arg\max_{y \in \mathcal{Y}^\mathcal{V}} \langle \theta, \phi(y) \rangle$$
$$\Rightarrow \ \exists c \in \mathcal{C}: \ \max_{y \in \mathcal{Y}} \theta^*_{c,y} > 0$$

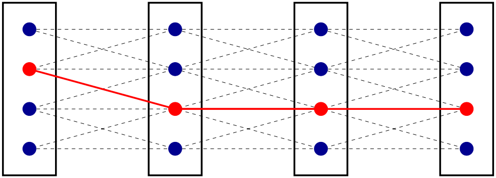
Implementation: Numerical Issues



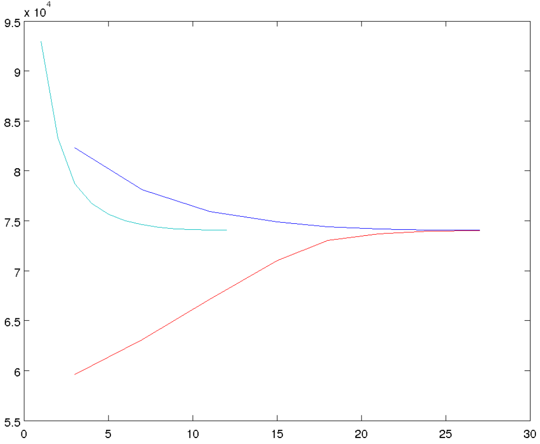
$$\theta^*_{c,y} = \theta_{c,y} - \theta_{c,y^*}$$
$$y^* = \arg\max_{y \in \mathcal{Y}^{\mathcal{V}}} \langle \theta, \phi(y) \rangle$$
$$\Rightarrow \; \exists c \in \mathcal{C}: \; \max_{y \in \mathcal{Y}} \theta^*_{c,y} > 0$$

Solution: an equivalent transformation $\theta \to \theta^*$:

$$\langle \theta, \phi(y) \rangle = \langle \theta^*, \phi(y) \rangle \;\; \forall y \in \mathcal{Y}^{\mathcal{V}}$$
$$y^* \in \operatorname{Arg}\max_{y \in \mathcal{Y}^{\mathcal{V}}} \langle \theta^*, \phi(y) \rangle \Leftrightarrow \forall c \in \mathcal{C}: \;\; \theta^*_{c,y^*} \in \operatorname{Arg}\max_{y \in \mathcal{Y}} \theta^*_{c,y}$$

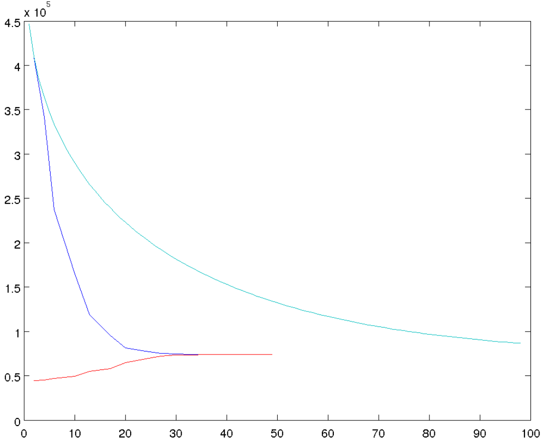


Random Sample: 5 Labels, an LP Tight Case. Close to a Trivial Problem



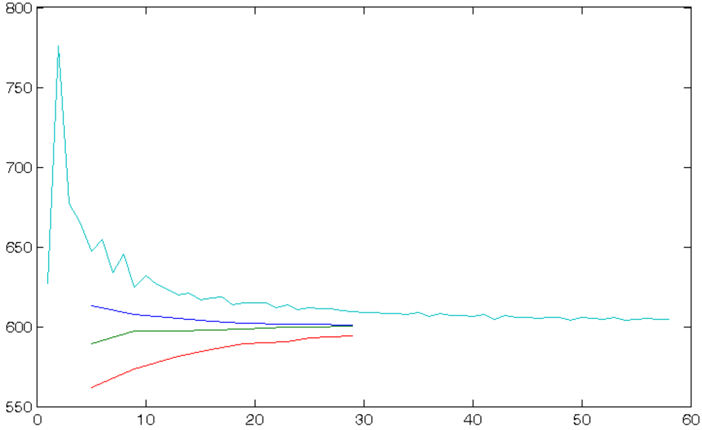
Checked on different image size from 20×20 to 200×200 and different distribution of weights between pairwise and unary factors.

Random Sample: 5 Labels, an LP Tight Case. Far from a Trivial Problem

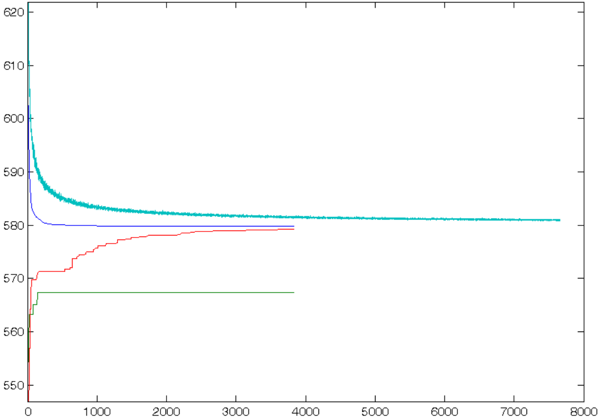


Checked on different image size from 20×20 to 200×200 and different distribution of weights between pairwise and unary factors.

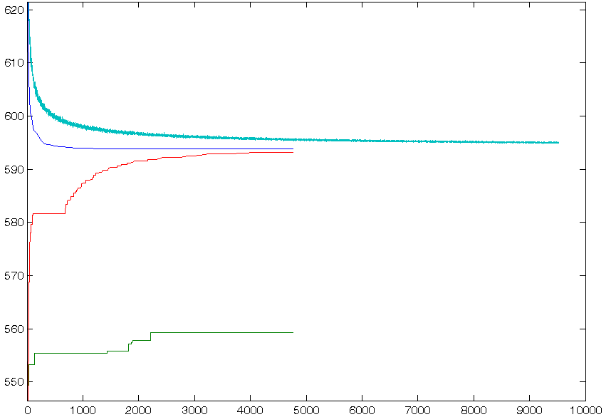
Random Sample: 5 Labels, Uniform Distribution with pw/u Weights 0.01



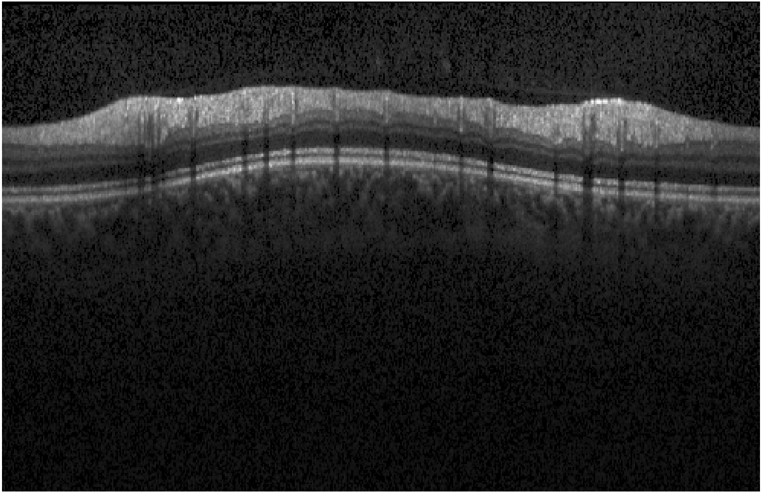
Random Sample: 5 Labels, Uniform Distribution with pw/u Weights 0.50



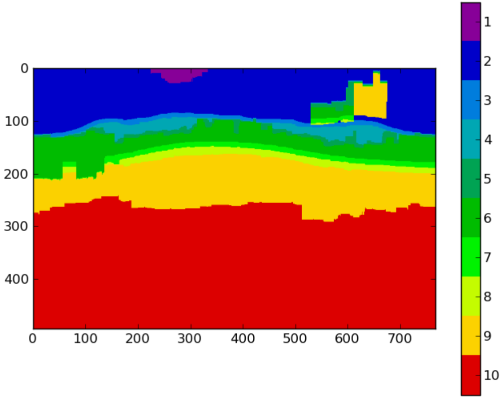
Random Sample: 5 Labels, Uniform Distribution with pw/u Weights 0.75



Input image



Precision 0.01% (227 oracle calls)



Summary

We proposed:

1. A smooth optimal first-order optimization method to solve the MAP inference problem.
2. A method for the lower bound calculation for non-LP-tight case.
3. Dynamic Lipschitz constant estimation to significantly speed-up the calculations.
4. A non-trivial implementation for (almost) any arbitrary small smoothing value.

Future work

1. Further speed-up.
2. Deeper insight to a way of changing the smoothing parameter in the course of the algorithm.
3. Improvement of convergence of the primal objective for a non-LP-tight cases.

Analysis of paper by Jojic, Gould, Koller: ICML 2010

+ Clever idea!
- Lipschitz constant is estimated for L_1-norm and algorithm (seems that) uses L_2-norm - error !
- Stopping criterion is not specified.
- Numerical issues are not covered: experiments only with decompositions to small subgraphs.

Optimization: Estimate Sequence Notion [Nesterov83]

1. $\{g^t(x)\}, \alpha^t \in (0,1),\ \sum_{t=0}^{\infty} = \infty, \quad t = \overline{1,\infty}$

$$g^{t+1}(x) \leqslant \alpha^t f(x) + (1-\alpha^t) g^t(x)$$

2. if for some $\{x^t\}$ $f(x^t) \leqslant g^{t*} \equiv \min_{x \in \mathbb{R}^n} g^t(x)$ then

$$f(x^t) - f^* \leqslant \lambda^t (g_0(x^t) - f^*) \to 0, \text{for } \lambda^t = \prod_{t=0}^{\infty} (1-\alpha^t) \to 0.$$

Optimization: Estimate Sequence Example [Nesterov83]

1. f-convex $\Rightarrow$ $f(x) \geqslant f(z) + \langle f'(z), x - z \rangle \Rightarrow$ for any sequence $\{z^t\}$

$$g^{t+1}(x) = \alpha^t (f(z) + \langle f'(z^t), x - z \rangle) + (1 - \alpha^t) g^t(x)$$

2. let $g_0(x) = g_0^* + \frac{\gamma_0}{2} ||x - u_0||^2$ then

$$g^t(x) = g^{t*} + \frac{\gamma^t}{2} ||x - u^t||^2$$

and for g^{t*}, γ^t, u^t there are closed form expressions.

3. We have to define sequences a^t, z^t and x^t such that $f(x^t) \leqslant g^{t*}$.
4.

$$f(x^t) - f^* \leqslant \lambda^t \left(f(x_0) - f^* + \frac{\gamma_0}{2} ||x_0 - x^*||^2 \right) \rightarrow 0$$

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