Abstract

- We consider a linear programming relaxation of the MAP-inference problem. Its dual can be treated as an unconstrained, concave but non-smooth one.
- We utilize smoothing and coordinate descent algorithm (smoothed TRW-S) to deal with the smoothed problem.
- We propose a smoothing scheme to adaptively decrease the smoothing degree. The scheme is based on an estimated duality gap.
- The method shows superior results on Middlebury benchmark.

Problem Formulation

Given the graph $G = (V, E)$, associated variables $x_u \in \mathbb{R}$, $w \in V \cup \mathcal{E}$, and potentials $\theta_{w,x_u} \in \mathbb{R}$, we consider the energy minimization problem

$$\min_{x \in \mathcal{X}} E(x) = \sum_{u \in V} \theta_{w,x_u} x_u + \sum_{v \in E} \sum_{u \in \mathcal{X}} \theta_{w,x_u,x_v} x_u x_v.$$  \hspace{1cm} (1)

Dual Decomposition and LP Relaxation

Let $F(D)$ be an updating rule for $F$ and $\Delta \lambda$ be a target solution accuracy. Initialize: $\lambda^0 \in \mathbb{R}^{|V|}$, $t = 0$, $E_{\text{min}}^0 = E(F(\lambda^0))$.

for $t = 0, \ldots, N$ do

if $E_{\text{min}}^t - U(\lambda^t) \leq \epsilon$ break

$\lambda^t \leftarrow \min \left\{ \lambda^* : F(\lambda) = U(\lambda) \right\}$. \hspace{1cm} # check stopping condition

Update $\lambda^t$ to $\lambda^{t+1}$. \hspace{1cm} # monotonic update of smoothing

Update $E_{\text{min}}^{t+1}$ by $E_{\text{min}}^{t+1} \leftarrow E_{\text{min}}^t$. \hspace{1cm} # update upper bound from gradients

end for

Experiments: Plots

- Left: venus. Right: computer generated 256 × 256, 4 labels grid model, potentials uniformly distributed on [0, 1].

Experiments: Table

Table: OC 0% and OC 1 – number of oracle calls required to achieve the relative precision 0.1% and the absolute precision 1 respectively.

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