MPLP++: Fast, Parallel Dual Block-Coordinate Ascent (BCA) for Dense Graphical Models

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Objective

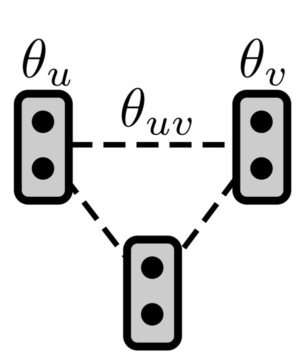
What:

- Fast and parallel algorithms for dense graphical models

Why:

- Dense graphical models are more expressive
- CNN+CRF training
- Huge datasets and Real Time Applications

MAP Inference



$$y^* = \arg\min_{y \in \mathcal{Y}^{\mathcal{V}}} \left[E(y|\theta) := \sum_{v \in \mathcal{V}} \theta_v(y_v) + \sum_{uv \in \mathcal{E}} \theta_{uv}(y_{uv}) \right]$$

- $\theta_v \to$ node potential.
- $\theta_{uv} \to$ pairwise potential.
- $y^* \to$ optimal labelling.

Dual LP

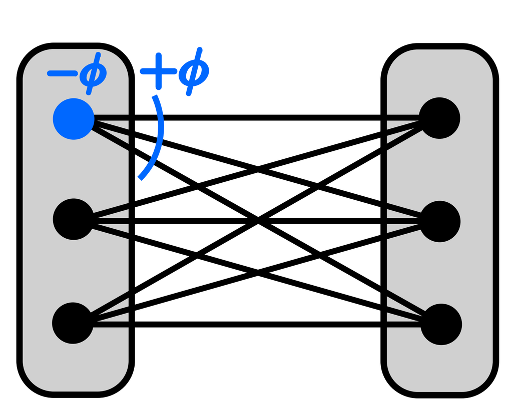
To be able to deal with arbitrary potentials, we address the dual problem:

$$D(\phi) := \sum_{u \in \mathcal{V}} \min_{s \in \mathcal{Y}} \theta^{\phi}_u(s) + \sum_{uv \in \mathcal{E}} \min_{(s,t) \in \mathcal{Y}^2} \theta^{\phi}_{uv}(s,t)\,. \qquad (1)$$

$$\theta^{\phi}_u(s) := \theta_u(s) + \textstyle\sum_{v \in \mathrm{Nb}(u)} \phi_{v \to u}(s)$$

$$\theta^{\phi}_{uv}(s,t) := \theta_{uv}(s,t) - \phi_{v \to u}(s) - \phi_{u \to v}(t)\,.$$

Dual variables $\phi_{u \to v}$ and $\phi_{v \to u}$ are the Lagrange multipliers.



$D(\phi)$ is

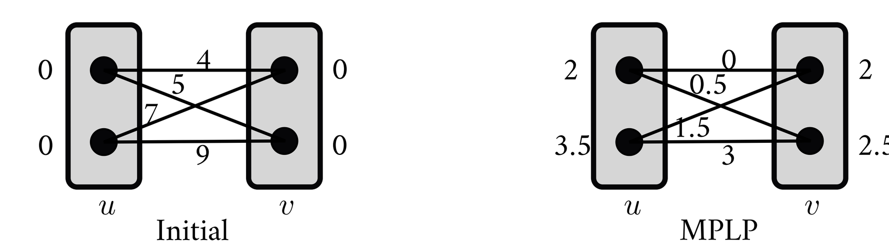
- concave
- piece-wise linear
- non-smooth

Motivation for Algorithm Structure

- The subgradient for the dual is sparse. Block Coordinate Ascent methods are more efficient than sub-gradient based methods.
- TRWS [1] is the best performer for MAP inference.
- Dense graphs need no large sub-problem decompositions.

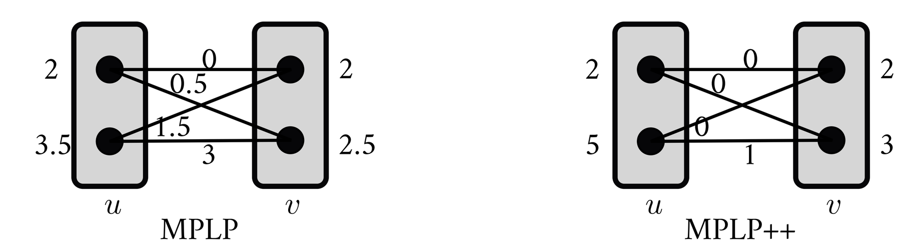
Comparing BCA Updates

$$g_{uv}(s,t) = \theta_{uv}(s,t) + \theta_u(s) + \theta_v(t)$$



MPLP: [2]

$$\begin{aligned} \theta^{\mathcal{M}}_u(s) &:= \tfrac{1}{2}\min_{t \in \mathcal{Y}} g_{uv}(s,t),\ \forall s \in \mathcal{Y}, \\ \theta^{\mathcal{M}}_v(t) &:= \tfrac{1}{2}\min_{s \in \mathcal{Y}} g_{uv}(s,t),\ \forall t \in \mathcal{Y}. \end{aligned} \qquad (\mathcal{M})$$



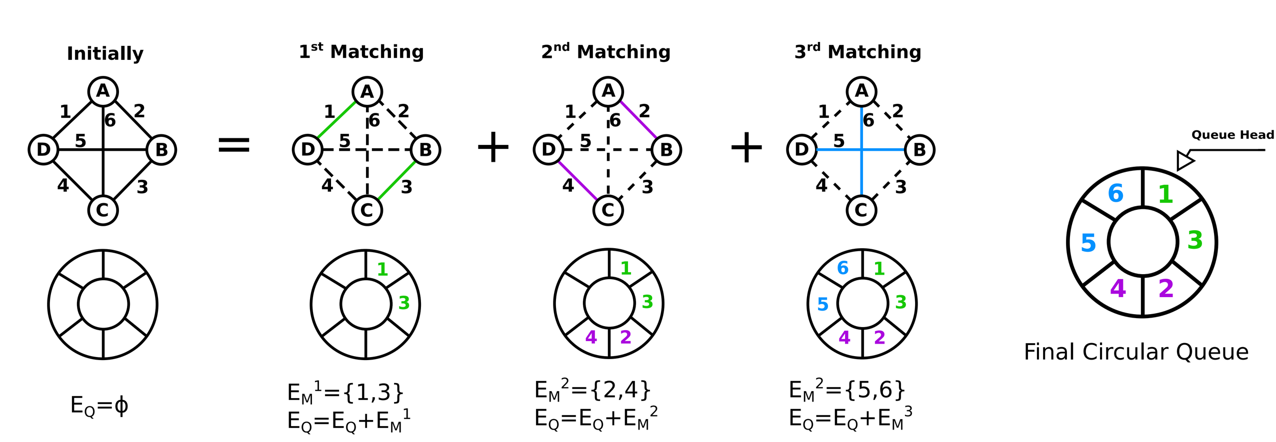
MPLP++:

$$\begin{aligned} &\theta^{\mathcal{H}}_u(s) := \theta^{\mathcal{M}}_u(s), \quad \theta^{\mathcal{H}}_v(s) := \theta^{\mathcal{M}}_v(s),\ \forall s \in \mathcal{Y}\,, \\ &\theta^{\mathcal{H}}_v(t) := \theta^{\mathcal{H}}_v(t) + \min_{s \in \mathcal{Y}}[g_{uv}(s,t) - \theta^{\mathcal{H}}_v(t) - \theta^{\mathcal{H}}_u(s)],\ \forall t \in \mathcal{Y}\,, \\ &\theta^{\mathcal{H}}_u(s) := \theta^{\mathcal{H}}_u(s) + \min_{t \in \mathcal{Y}}[g_{uv}(s,t) - \theta^{\mathcal{H}}_v(t) - \theta^{\mathcal{H}}_u(s)],\ \forall s \in \mathcal{Y}\,. \end{aligned} \qquad (\mathcal{H})$$

We prove

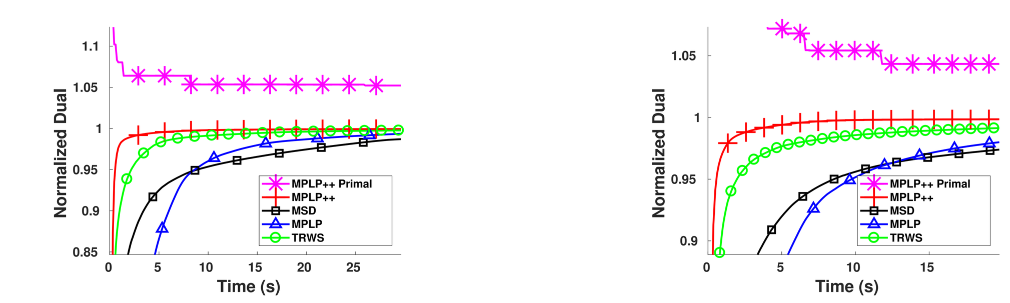
With same input, at the end of the first iteration, $D(\phi^{\mathcal{M}}) \le D(\phi^{\mathcal{H}})$.

Parallelization: CPU & GPU



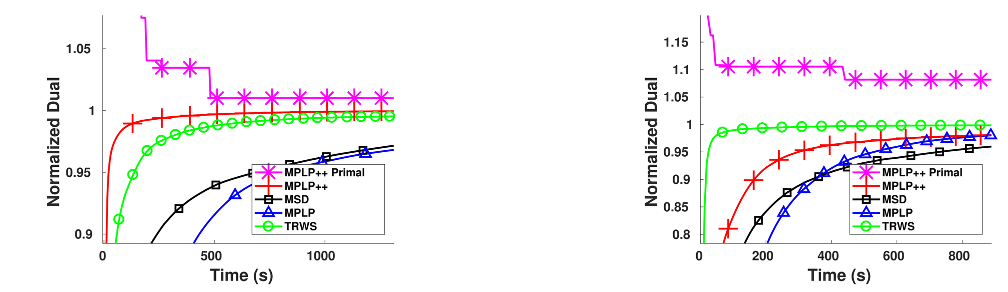
Non-incident edges can be processed in parallel. $\to$ We use maximum matching solvers.

Results



(e) protein (100% density, 40 vars, up to 200 states)

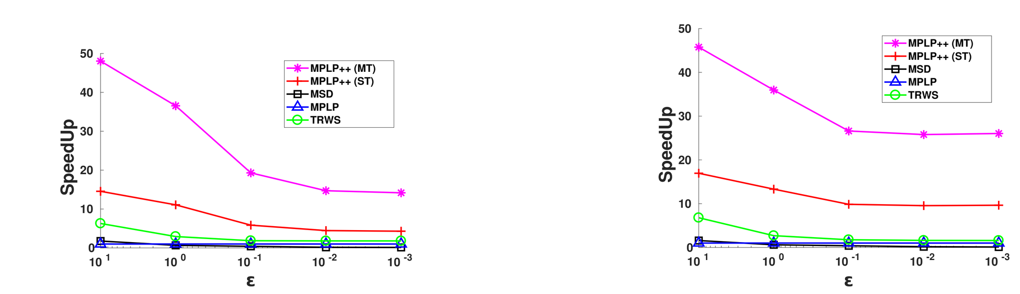
(f) worms (10% density, 556 vars, 40-50 states)



(g) pose (100% density, 600-4800 nodes,13 states)

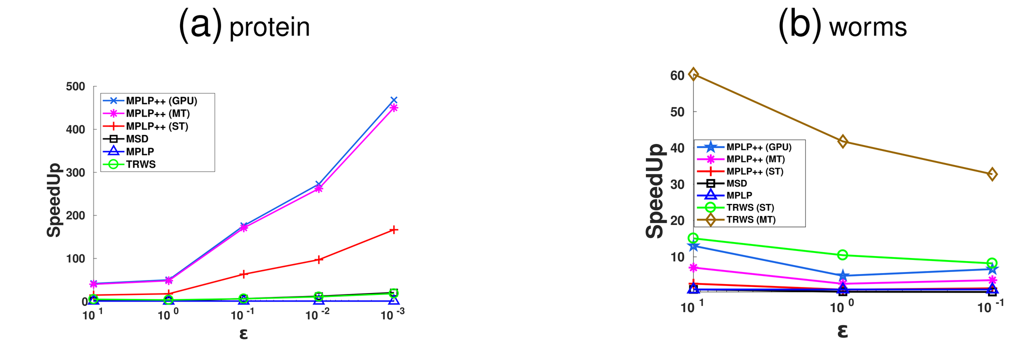
(h) stereo (tsukuba, venus, teddy, grid graph, 12-60 states)

Figure 1: Dual versus time



(a) protein

(b) worms



(c) pose

(d) stereo

Figure 2: Obtained Speedups

Conclusions and Outlook

- Our approach is the state-of-the-art method for dense graphical models ($>10\%$ graph density), beating even TRWS.
- We give CPU and GPU parallel implementations provided at **XXXX**.

References

[1] Kolmogorov, V., "Convergent tree-reweighted message passing for energy minimization", PAMI, 2006.

[2] Globerson et al., "Fixing Max-Product: Convergent Message Passing Algorithms for MAP LP-Relaxations", NIPS, 2008.

[3] A. Shekhovtsov, C. Reinbacher, G.Graber and T. Pock: Solving Dense Image Matching in Real-Time Using Discrete-Continuous Optimization, CVWW 2016.

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