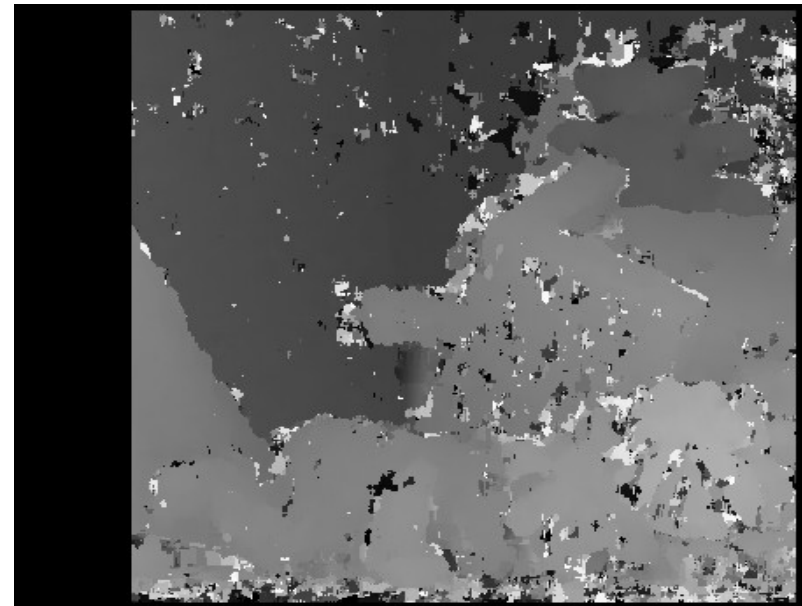


# Computer Vision I

## Dense Stereo Correspondences

# A Basic Stereo Algorithm: Blockmatching

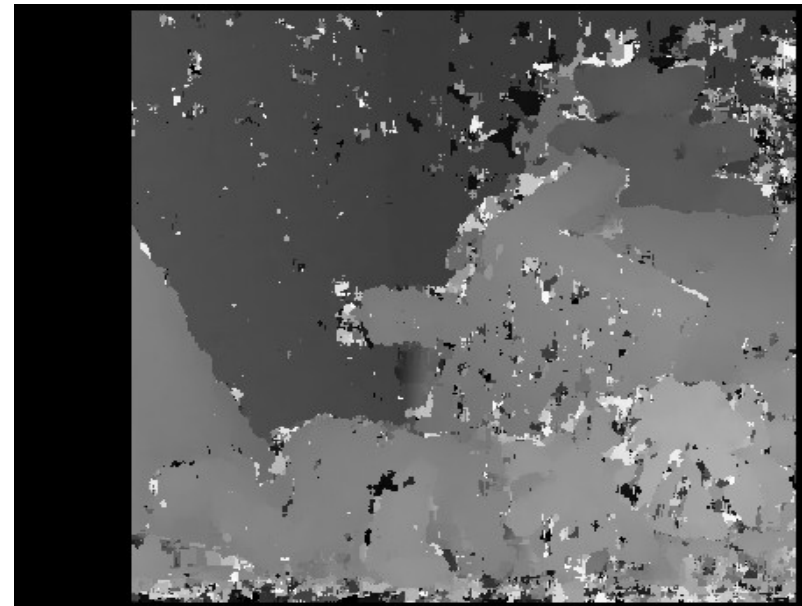
- For each pixel
  - For each disparity value
    - Compare patch similarity
    - Assign disparity with highest similarity



Result: Noisy, no smooth surfaces

# A Basic Stereo Algorithm: Blockmatching

$$\forall (x, y) \in \Omega \quad \hat{d}(x, y) = \operatorname{argmin}_{d \in D} C(x, y, d)$$



Result: Noisy, no smooth surfaces

# Stereo Algorithm: Sanity Check

- Uniqueness constraint
  - Only one pixel should arrive at each target location





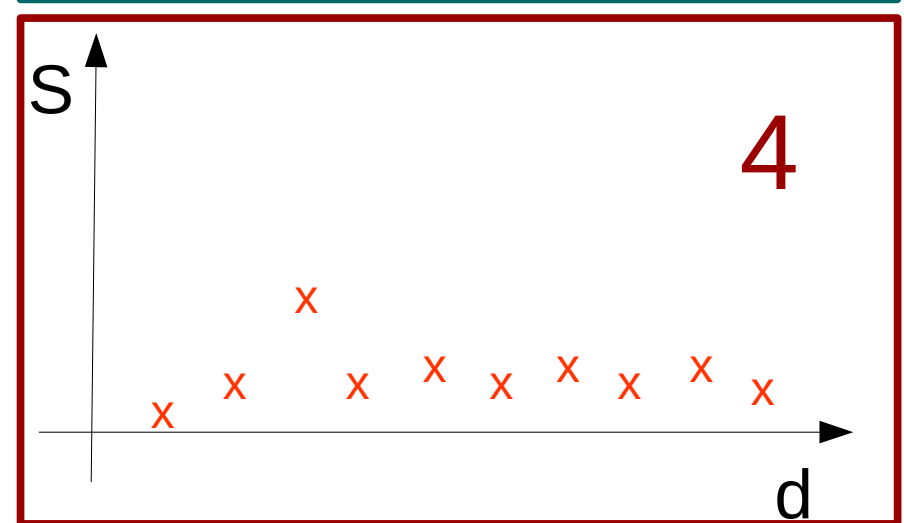
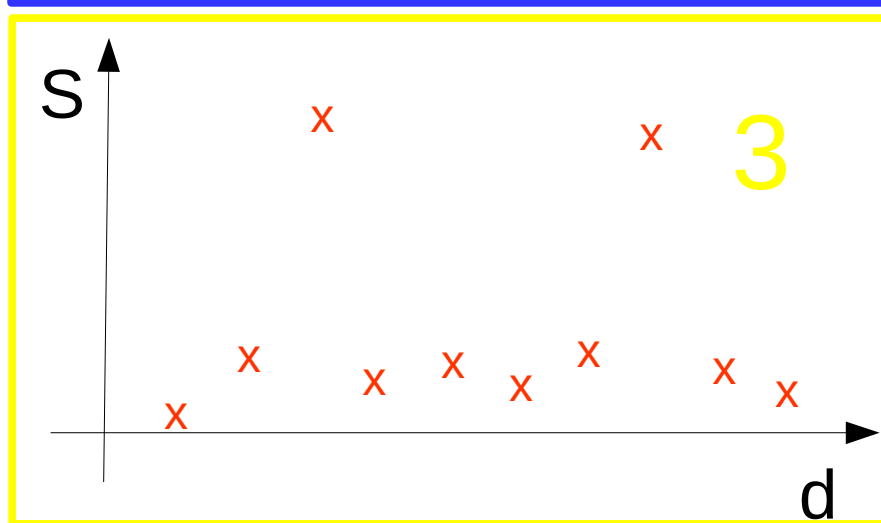
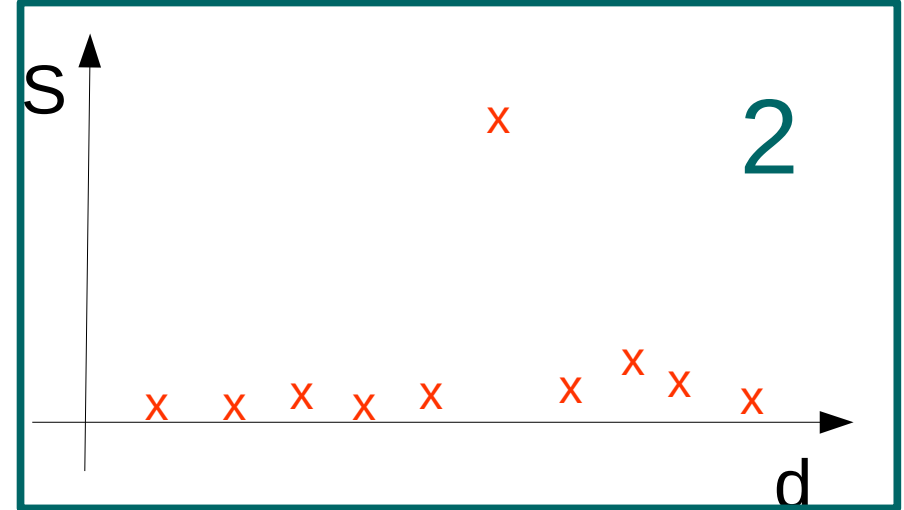
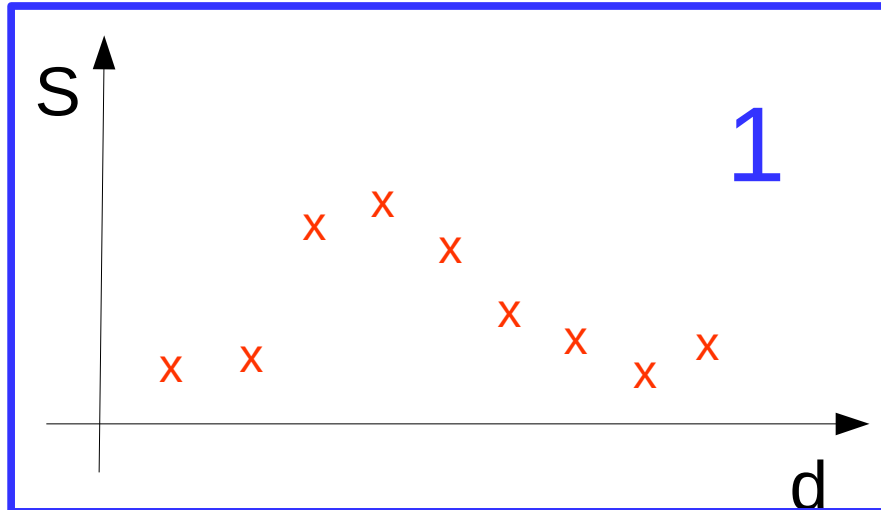
# Stereo Algorithm: Sanity Check

- Left-Right consistency
  - Compute disparity with left reference image  $d_l$
  - Compute disparity with right reference image  $d_r$

$$|d_l(p) - d_r(q)| > 1 \quad q = d_l(p)$$



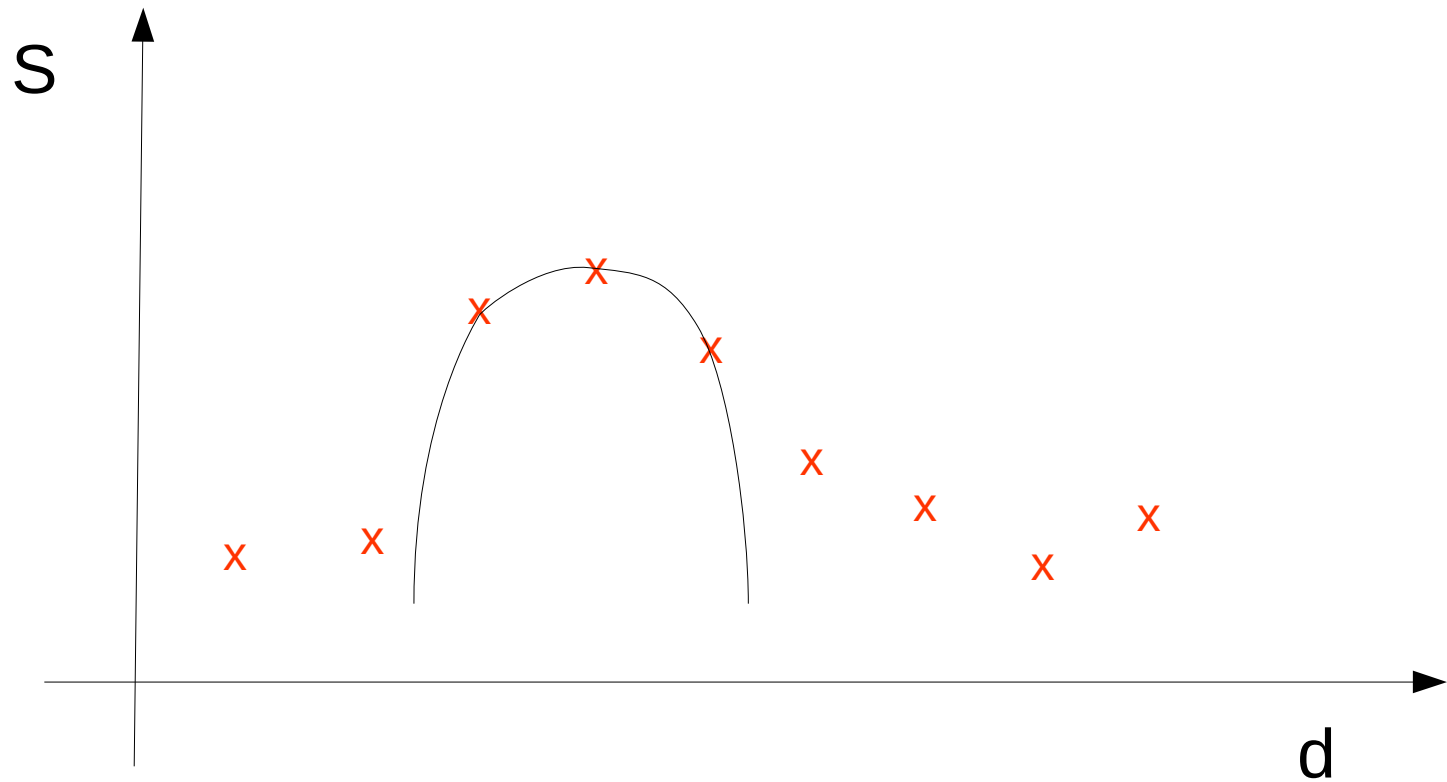
# Quiz: In which situation are you confident with your stereo correspondence?



5: None

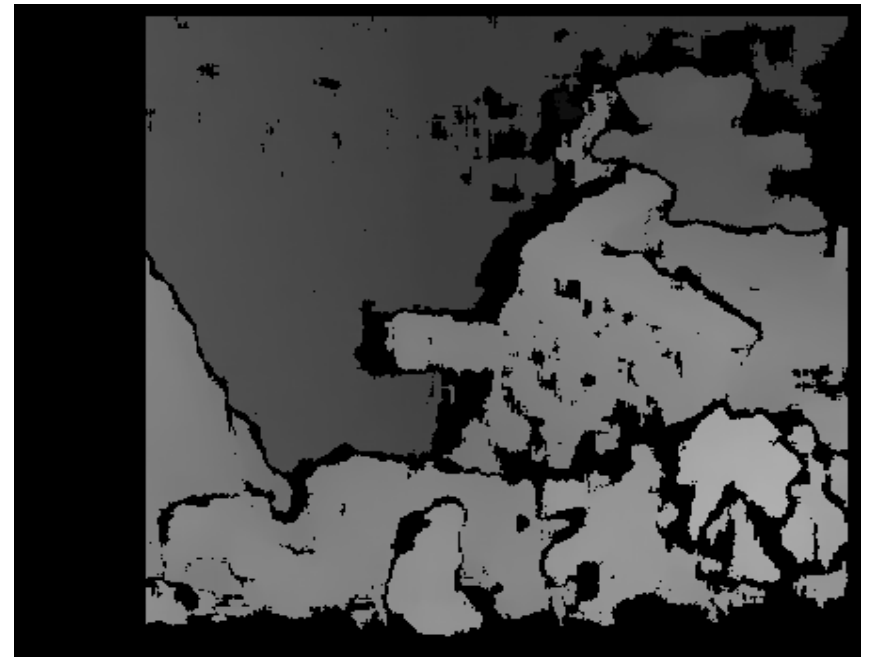
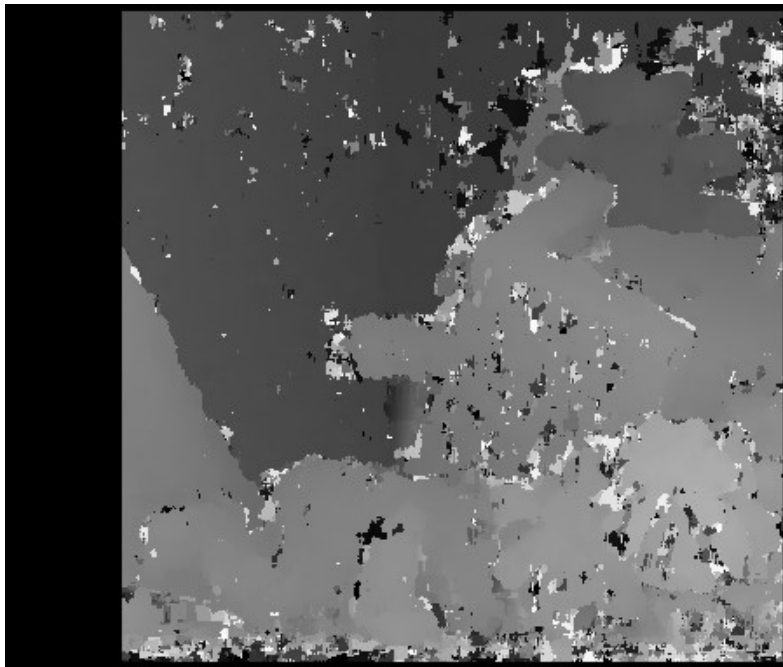
6: Don't know

# Disparity Refinement



# Stereo Algorithm: Sanity Check

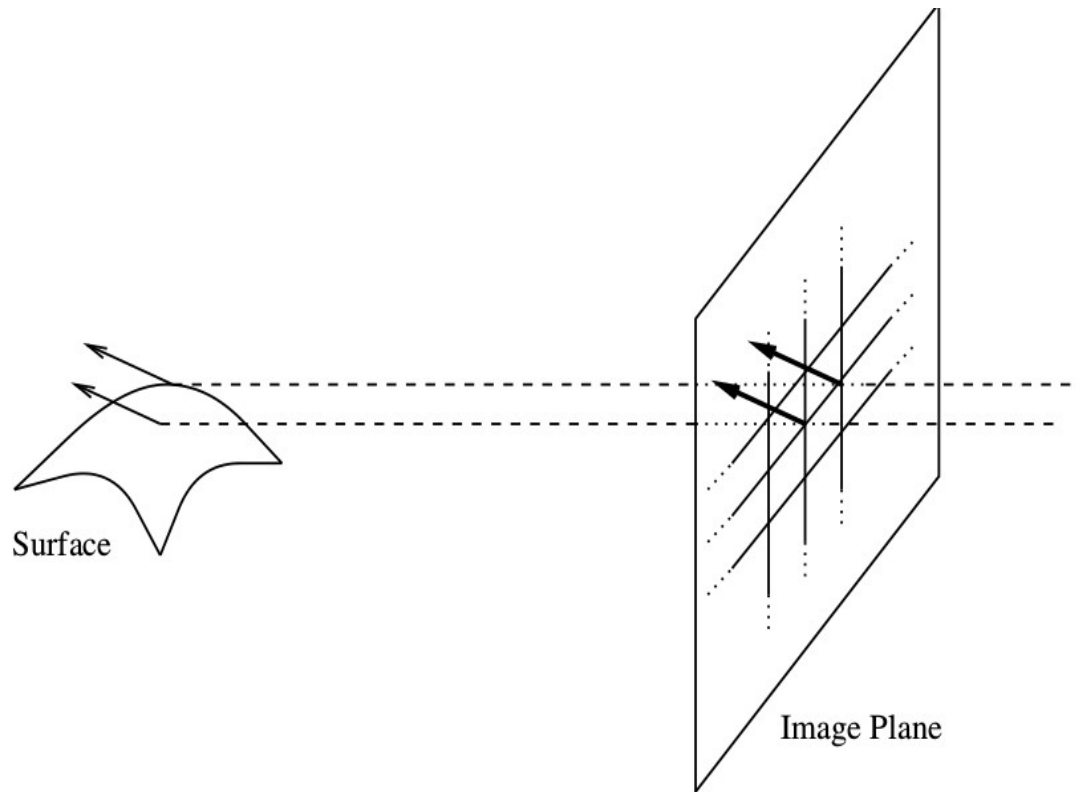
- Confidence considerations





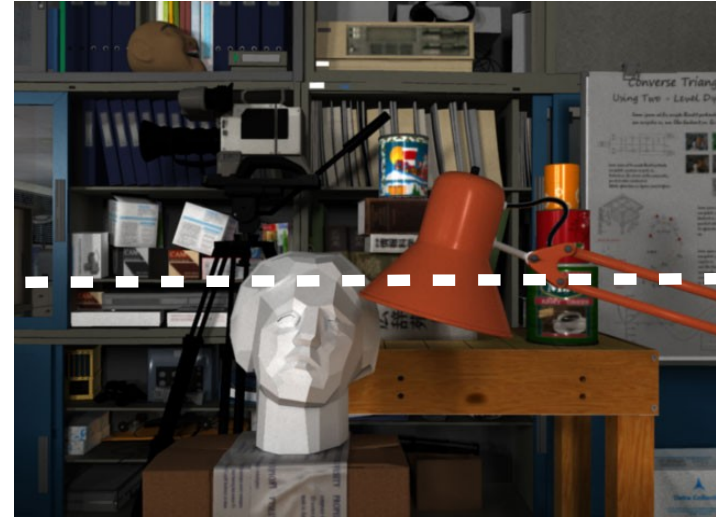
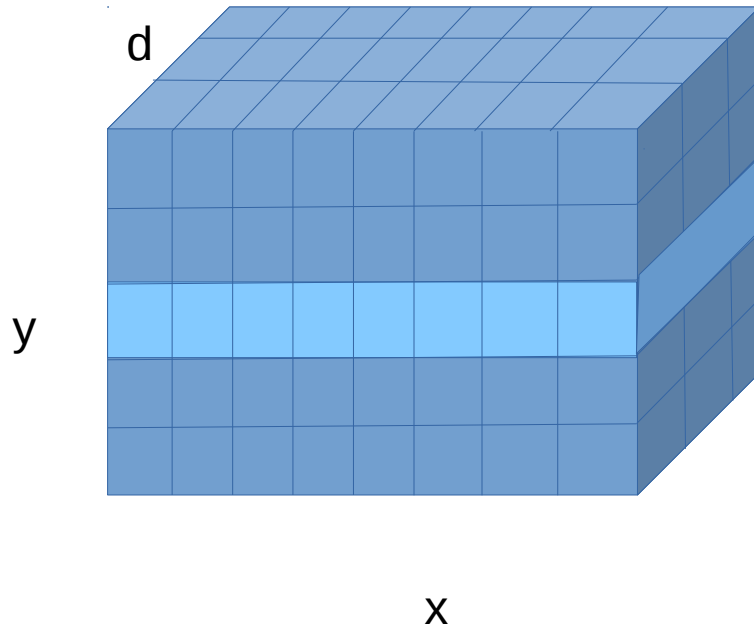
# Disparity Estimation: Assumption II

- Neighboring pixels belong to the same surface



$$d(x, y) \approx d(x + \epsilon_x, y + \epsilon_y)$$

# Disparity Space Image

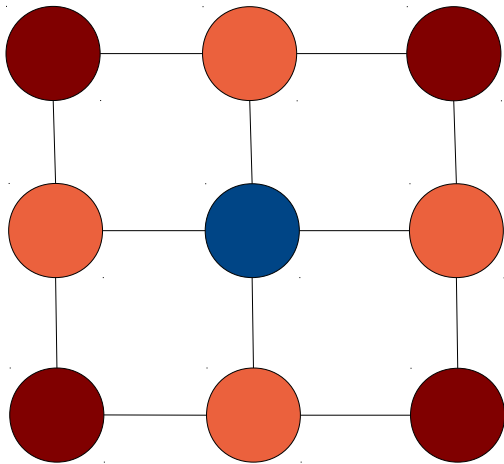


# Disparity Fields

- Neighboring pixel should have similar disparity

$$\hat{d}:\Omega\rightarrow D, \quad (x,y)\mapsto\hat{d}(x,y)$$

$$\hat{d}=\operatorname{argmin}_{d:\Omega\rightarrow D} \sum_{(x,y)\in\Omega} \left( C(x,y,d(x,y)) + \sum_{(s,t)\in N(x,y)} \rho(d(x,y),d(s,t)) \right)$$

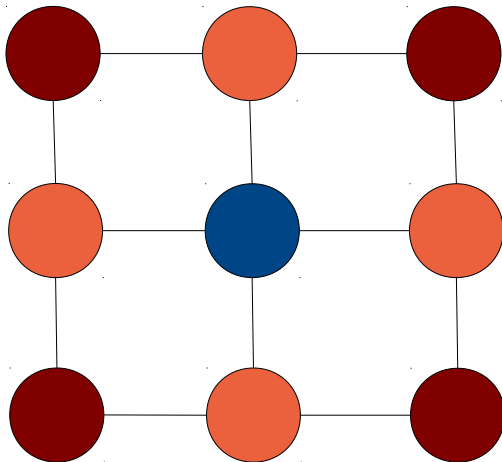


# Disparity Fields

- Neighboring pixel should have similar disparity

$$\hat{d}:\Omega\rightarrow D, (x,y)\rightarrow\hat{d}(x,y)$$

$$\hat{d}=\operatorname{argmin}_{d:\Omega\rightarrow D}\sum_{(x,y)\in\Omega}\left(C(x,y,d(x,y))+\sum_{(s,t)\in N(x,y)}\rho(d(x,y),d(s,t))\right)$$

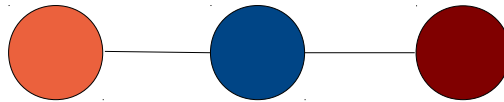


- Discrete Optimization Problem
- In general NP hard  $|\Omega|^{|D|}$

# Disparity Chains

- Neighboring pixel should have similar disparity
  - On an epipolar line

$$\hat{d} = \operatorname{argmin}_{d: \Omega \rightarrow D} \sum_{(x, y) \in \Omega} (C(x, y, d(x, y)) + \rho(d(x, y), d(x-1, y)))$$

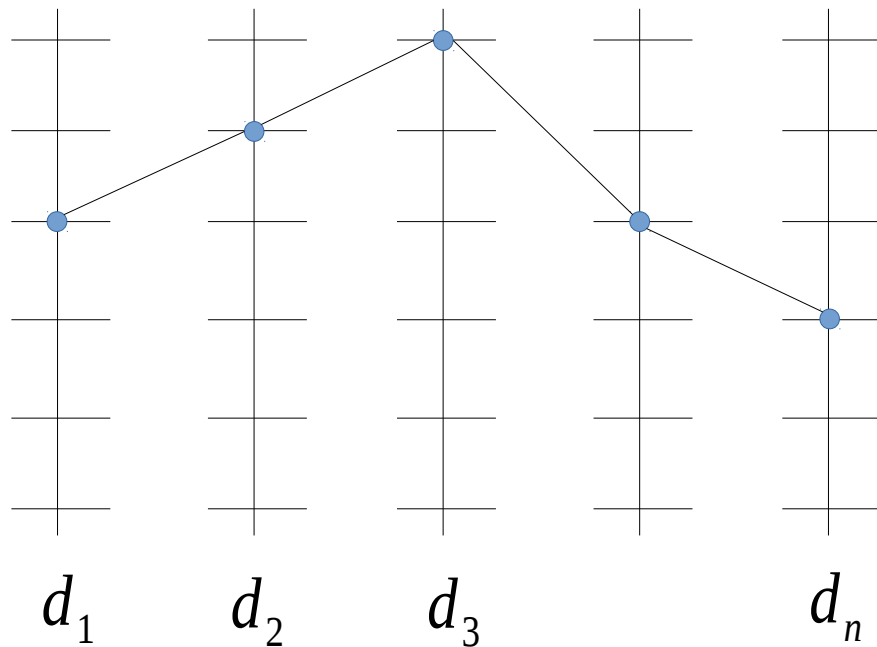




# Dynamic Programming

- Break the optimization down to sub-optimizations

$$E(\mathbf{x}) = \sum_{i=1}^n C(d_i) + \sum_{i=1}^n \rho(d_{i-1}, d_i)$$



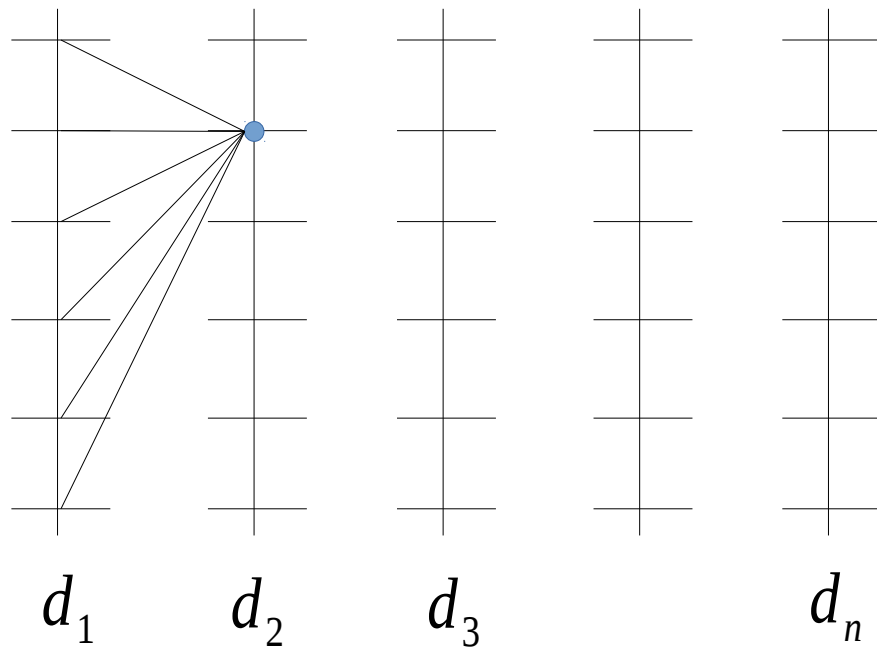
$$|\Omega|^{D|}$$

# Dynamic Programming

- Break the optimization down to sub-optimizations

$$S_2(d_2) = C(d_2) + \min_{d_1} \{ C(d_1) + \rho(d_1, d_2) \}$$

Remember  $d_1$  for which  $S_2(d_2)$  is a minimum



$$|\Omega|^{|\mathcal{D}|}$$

vs

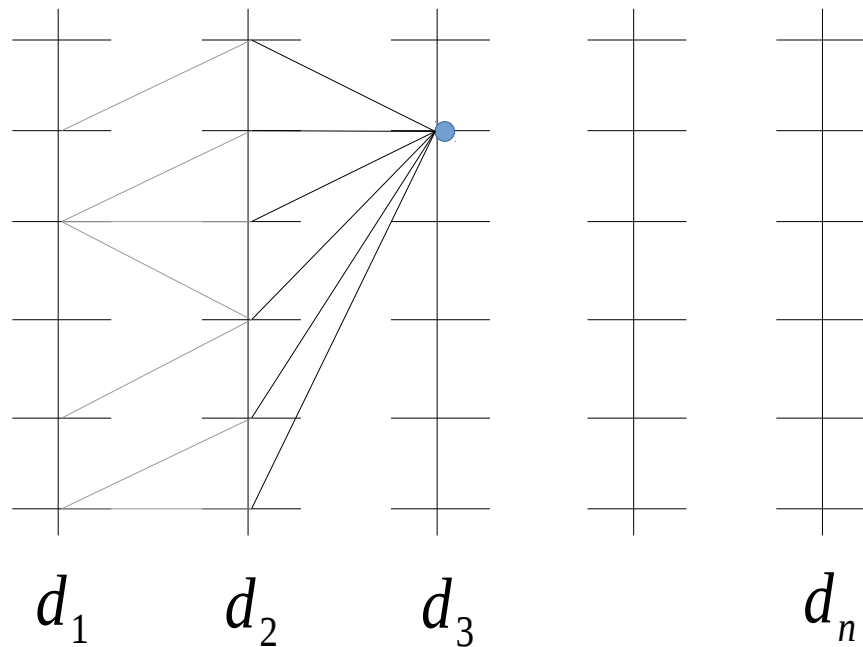
$$|D|^2 + \dots$$

# Dynamic Programming

- Break the optimization down to sub-optimizations

$$S_3(d_3) = C(d_3) + \min_{d_2} \{ S_2(d_2) + \rho(d_2, d_3) \}$$

Remember  $d_2$  for which  $S(d_3)$  is a minimum



$|\Omega|^{|D|}$

vs

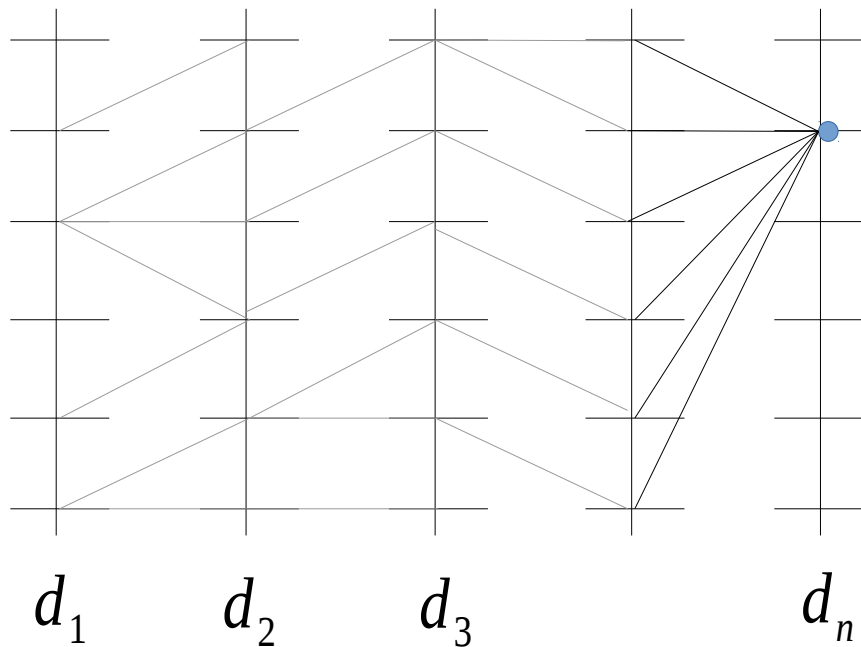
$2|D|^2 + \dots$

# Dynamic Programming

- Break the optimization down to sub-optimizations

$$S_n(d_n) = C(d_n) + \min_{d_{n-1}} \{ S_{n-1}(d_{n-1}) + \rho(d_{n-1}, d_n) \}$$

Remember  $d_{n-1}$  for which  $S_n(d_n)$  is a minimum



$$\hat{d}_n = \operatorname{argmin}_{d_n} S_n(d_n)$$

# Dynamic Programming Approach

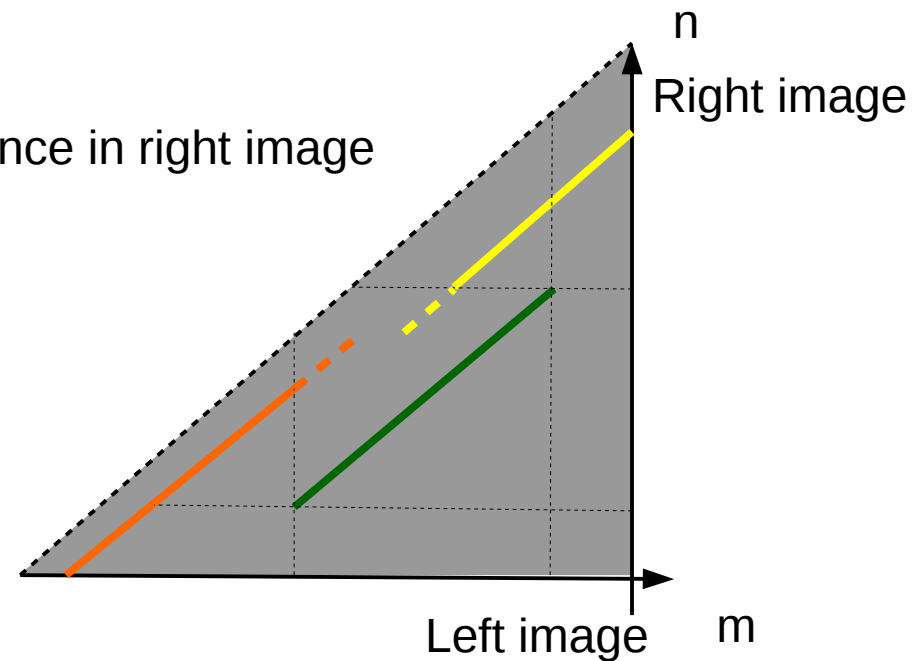
- Work on a single epipolar line



Pixels can at most have disparity 0

A point in the left image has a correspondence in right image with a smaller x-coordinate-index

Objects in front have larger disparity



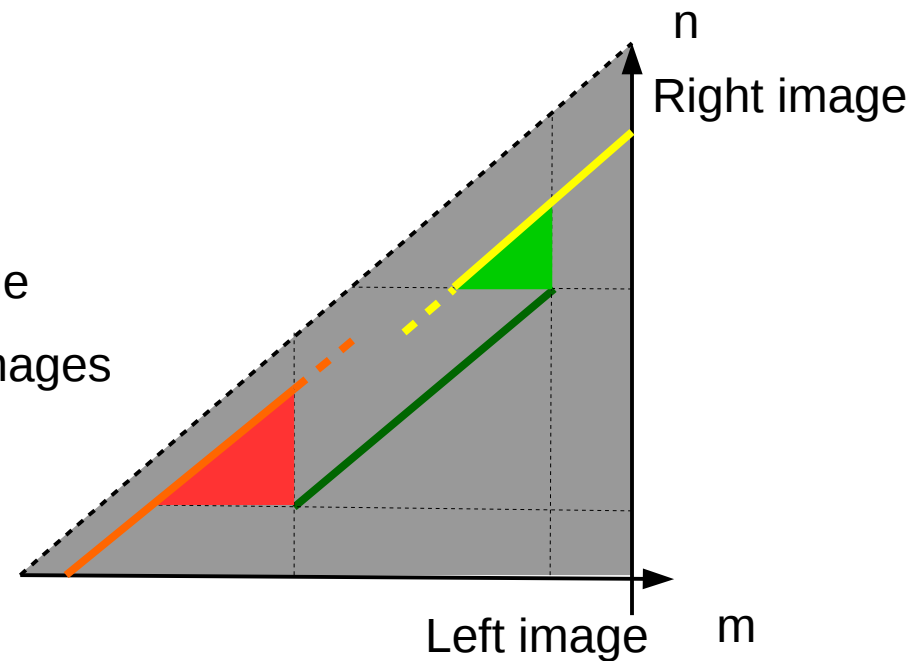
# Dynamic Programming Approach

- Work on a single epipolar line



Part of the scene is visible in only one image

Part of the scene is visible in none of the images



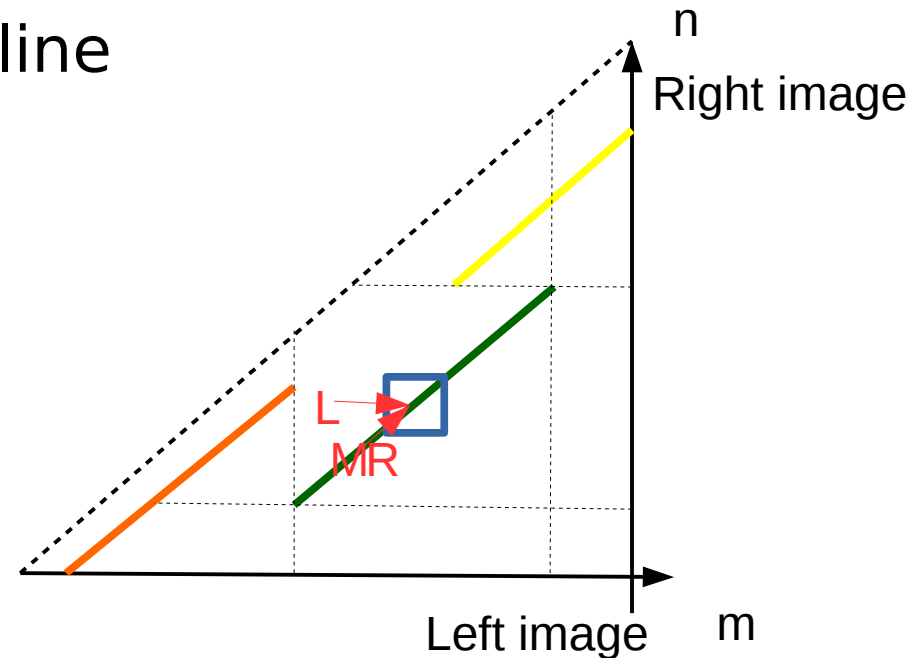
# Dynamic Programming Approach

- Work on a single epipolar line



- Design cost function with case distinction
  - Match

$$S(m, n, M) = \min(S(m-1, n-1, M), S(m-1, n, L), S(m-1, n-1, R)) + C(m, n)$$



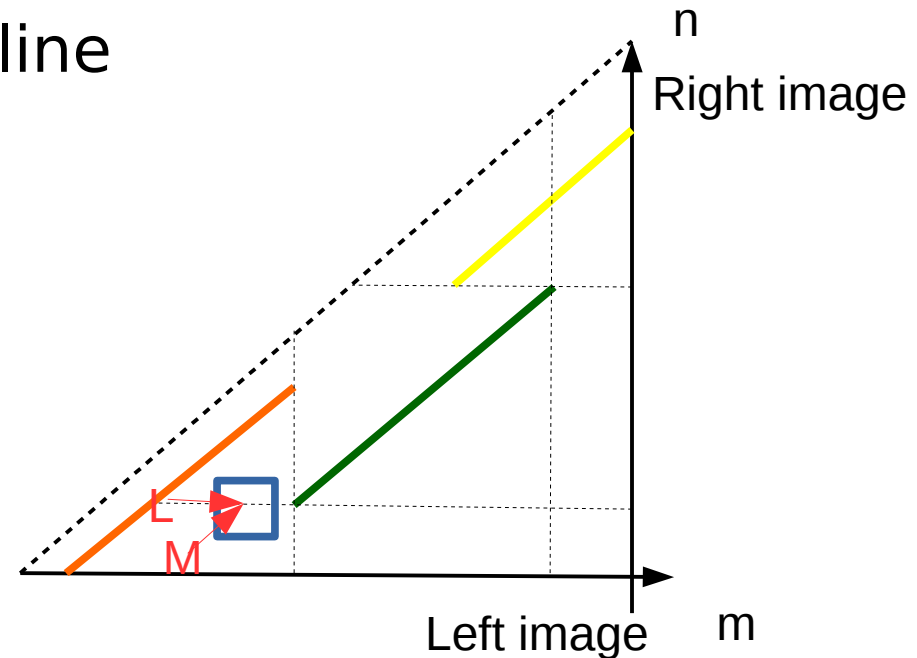


# Dynamic Programming Approach

- Work on a single epipolar line



- Recursive cost function
  - Left Occlusion



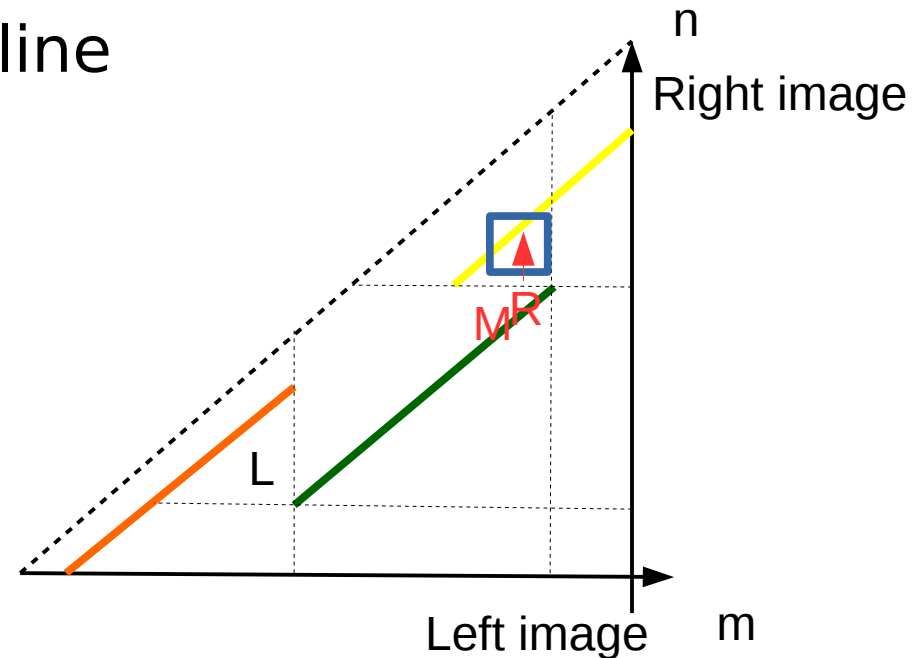
$$S(m, n, L) = \min(S(m-1, n-1, M), S(m-1, n, L)) + O$$

# Dynamic Programming Approach

- Work on a single epipolar line



- Recursive cost function
  - Right Occlusion



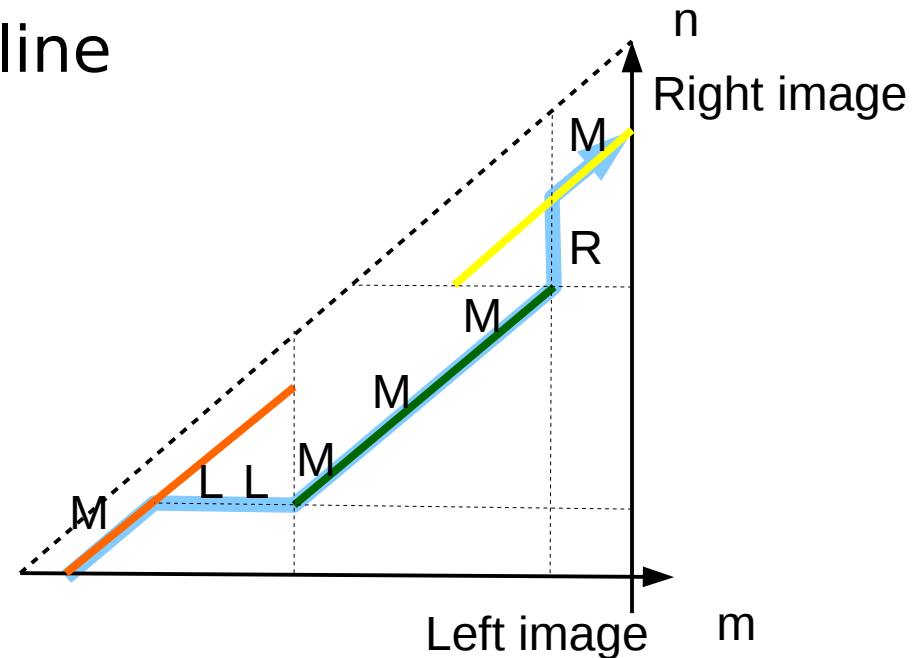
$$S(m, n, R) = \min(S(m, n-1, M), S(m, n-1, R)) + O$$

# Dynamic Programming Approach

- Work on a single epipolar line



- Recursive cost function
  - Ordering constraint:  
not all constellations are possible



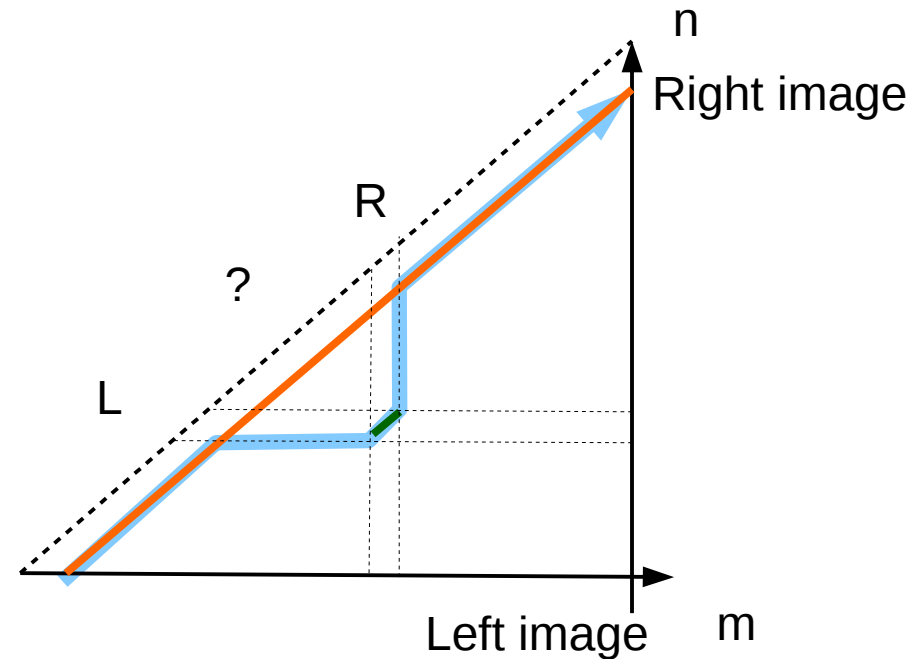
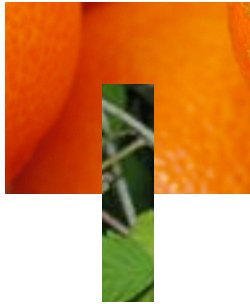
# Dynamic Programming

- Streaking Artifacts



# Dynamic Programming Approach

- Thin Objects

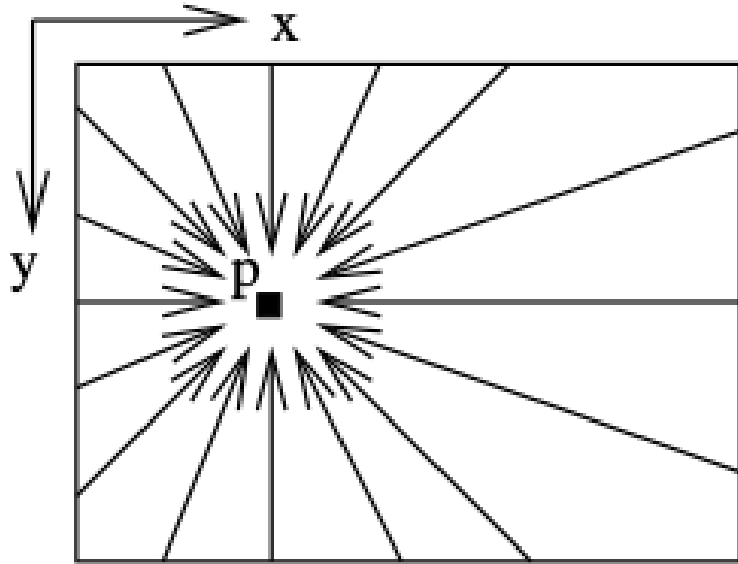


# Dynamic Programming

- Pros:
  - Can be optimized in polynomial time
  - Incorporates consistency and ordering constraint
- Cons:
  - Streaking artifacts
  - Thin objects that violate ordering constraint

# Semi Global Matching

- Idea: Consider more than one scanline



[ Hirschmüller (CVPR 2005): Accurate and efficient stereo processing by semi-global matching and mutual information]

# Semi Global Matching

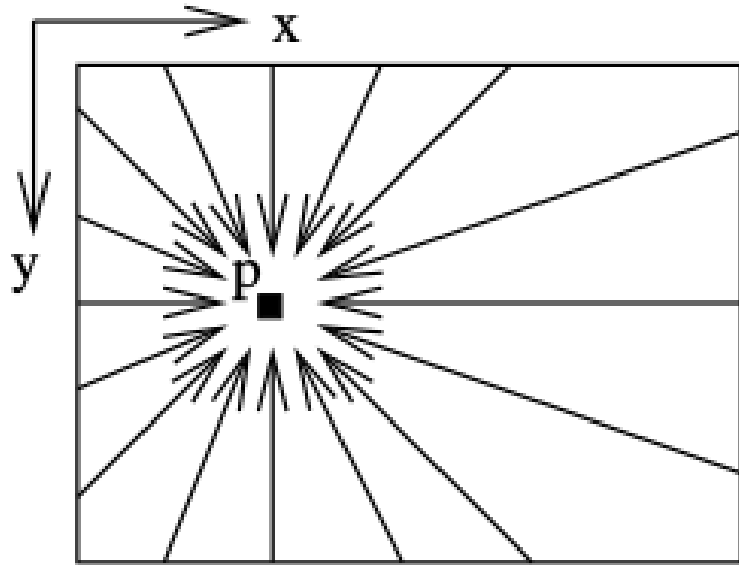
- Idea: Consider more than one scanline
- Drop ordering constraint and allow for all possible disparity changes

$$\rho(d_p, d_q) = \begin{cases} P_1 & \text{if } |d(p) - d(q)| = 1 \\ P_2 & \text{if } |d(p) - d(q)| > 1 \\ 0 & \text{else} \end{cases}$$

[ Hirschmüller (CVPR 2005): Accurate and efficient stereo processing by semi-global matching and mutual information]

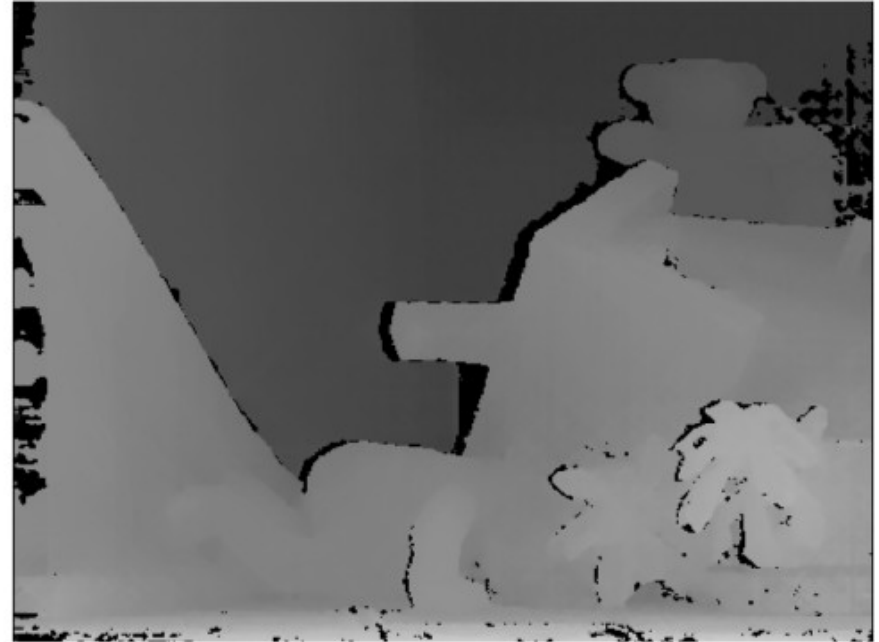


# Semi Global Matching



$$L_r(p, d) = C(p, d) + \min \begin{cases} L_r(p-r, d) \\ L_r(p-r, d-1) + P_1 \\ L_r(p-r, d+1) + P_1 \\ \min_i L_r(p-r, i) + P_2 \end{cases}$$

# Semi Global Matching



$$\hat{d}(p) = \operatorname{argmin}_r \sum L_r(p, d)$$

# A Global Stereo Algorithm

- Make the disparity of a pixel dependent on all other disparities!
  - Avoid streaking artifacts

$$\hat{d} = \operatorname{argmin}_{d: \Omega \rightarrow D} \sum_{(x,y) \in \Omega} \left( \underbrace{C(x,y,d(x,y))}_{\text{Pixel/ Patch Similarity}} + \sum_{(s,t) \in N(x,y)} \underbrace{\rho(d(x,y), d(s,t))}_{\text{Smoothness Prior}} \right)$$

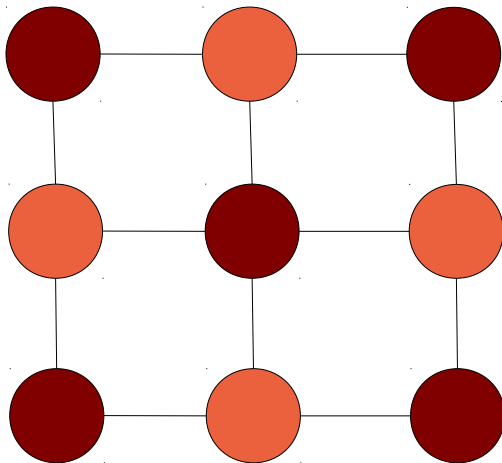
# A Global Stereo Algorithm

- Make the disparity of a pixel dependent on all other disparities!
  - Avoid streaking artifacts

$$\hat{d} = \underset{d: \Omega \rightarrow D}{\operatorname{argmin}} \sum_{(x,y) \in \Omega} \left( \underbrace{C(x,y,d(x,y))}_{\text{Pixel/ Patch Similarity}} + \sum_{(s,t) \in N(x,y)} \underbrace{\rho(d(x,y), d(s,t))}_{\text{Smoothness Prior}} \right)$$

Pixel/ Patch Similarity

Smoothness Prior



# A Global Stereo Algorithm

- Make the disparity of a pixel dependent on all other disparities!
  - Avoid streaking artifacts
  - NP-hard problem
  - Find approximate solution by using
    - Graph Cut algorithm
    - Belief Propagation

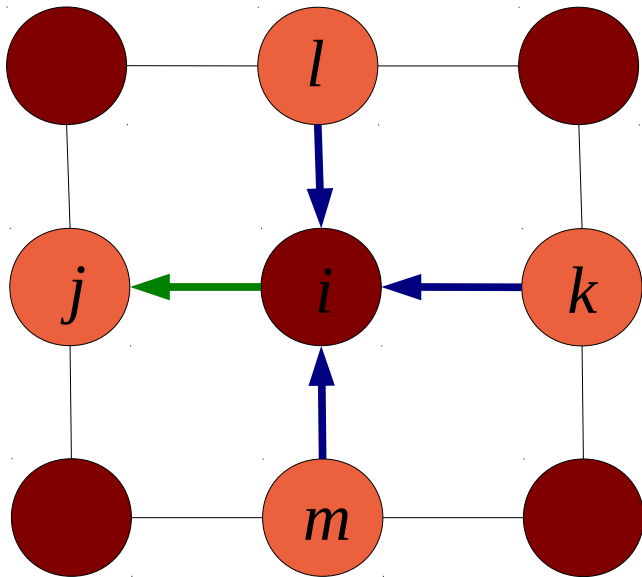
# Belief Propagation Approach

- Make the disparity of a pixel dependent on all other disparities!
  - Avoid streaking artifacts
  - NP-hard problem
  - Find approximate solution by using
    - Graph Cut algorithm
    - **Belief Propagation**

[ Tappen, Freeman: Comparison of Graph Cuts with Belief Propagation for Stereo, using Identical MRF Parameters]

# Belief Propagation Approach

What node  $i$  beliefs about each possible state of node  $j$

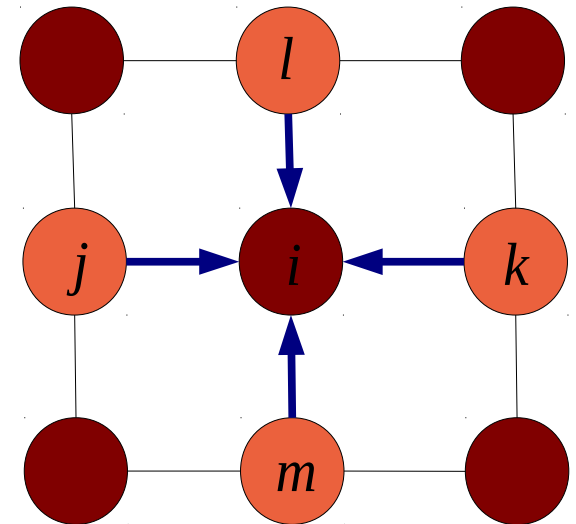


No message from  $j$ !

$$M_{i \rightarrow j}(d_j) = \min_{d_i} C(d_i) + \rho(d_i, d_j) + M_{l \rightarrow i}(d_i) + M_{k \rightarrow i}(d_i) + M_{m \rightarrow i}(d_i)$$

# Belief Propagation Schedule I

- Initialize all messages as 0
- Iterate
  - For each node: Compute all outgoing messages
  - Send messages
- Pick solution with min energy

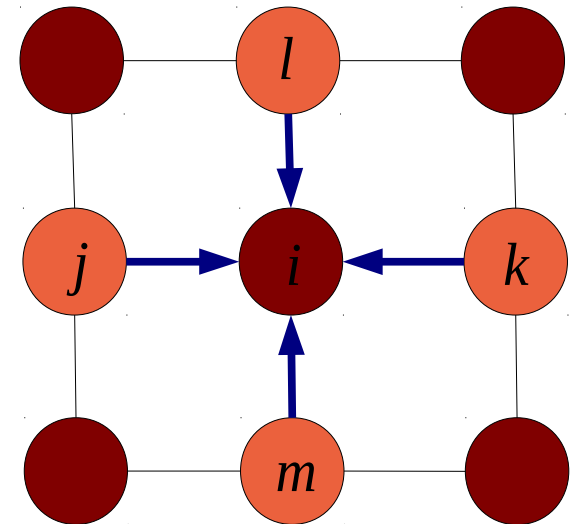


$$\hat{d}_i = \min_{d_i} C(d_i) + M_{j \rightarrow i}(d_i) + M_{l \rightarrow i}(d_i) + M_{k \rightarrow i}(d_i) + M_{m \rightarrow i}(d_i)$$



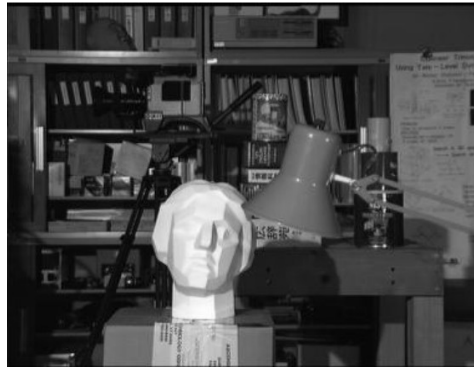
# Belief Propagation Schedule II

- Initialize all messages as 0
- Iterate (change proceeding direction)
  - For each node
    - Compute all outgoing messages
    - Send message
- Pick solution with min energy



$$\hat{d}_i = \min_{d_i} C(d_i) + M_{j \rightarrow i}(d_i) + M_{l \rightarrow i}(d_i) + M_{k \rightarrow i}(d_i) + M_{m \rightarrow i}(d_i)$$

# Belief Propagation



Semi Global Matching



Belief Propagation



# Belief Propagation Stereo

- Advantages
  - Global energy formulation
  - Arbitrary data/ smoothness term
- Disadvantages
  - Iterative
  - Approximative algorithm for energy

# Steps in Correspondence Estimation

1. Matching cost computation
2. Cost aggregation
3. Disparity computation / optimization
4. Disparity refinement



<http://vision.middlebury.edu/stereo/>

[ Scharstein, Szeliski (IJCV 2002): A taxonomy and evaluation of dense two-frame stereo correspondence algorithms]

# The Limits of Stereo Computation

- Brightness constancy per pixel
- or
- Brightness constancy on patch
  - Occlusion
  - Suitable Baseline
  - Texture



# Active Depth Cameras



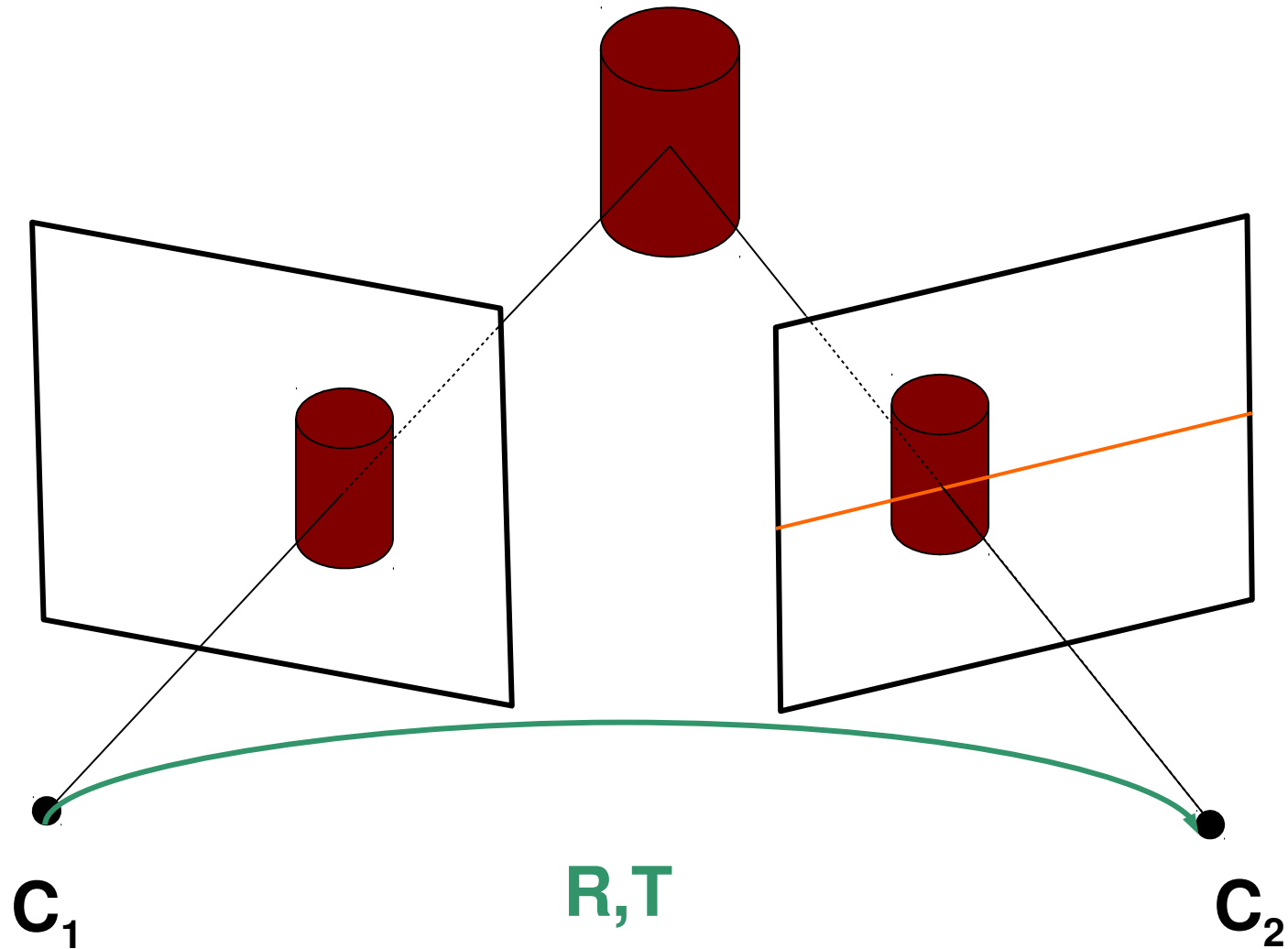
[Zeng (2012): Microsoft kinect sensor and its effect]

# Active Depth Cameras

- Impact on consumer market
  - 10 million units sold with first 3 months
- Impact on scientific world
  - Numerous publications with RGBD images
  - Increased interest in depth cameras

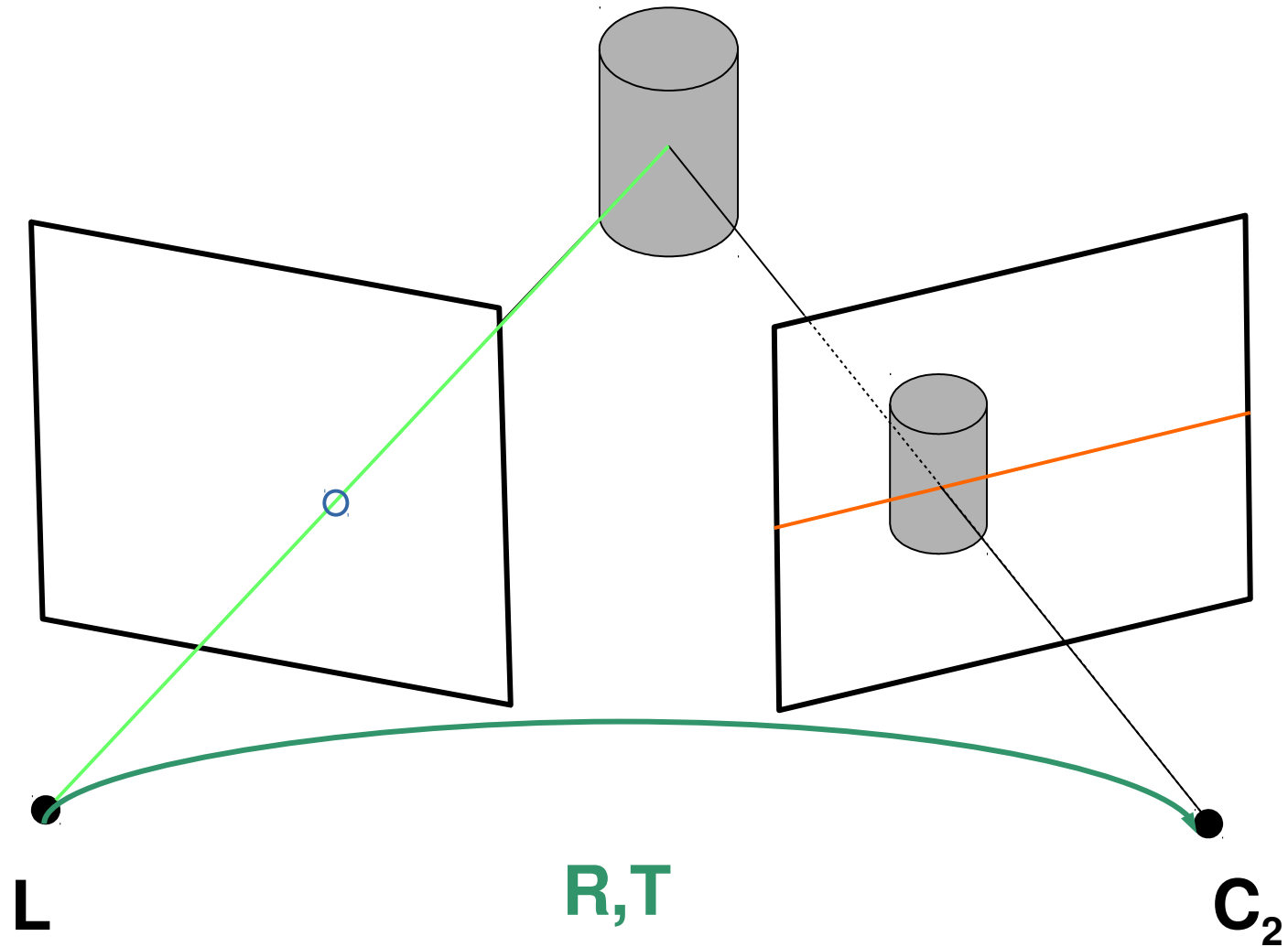


# Triangulation

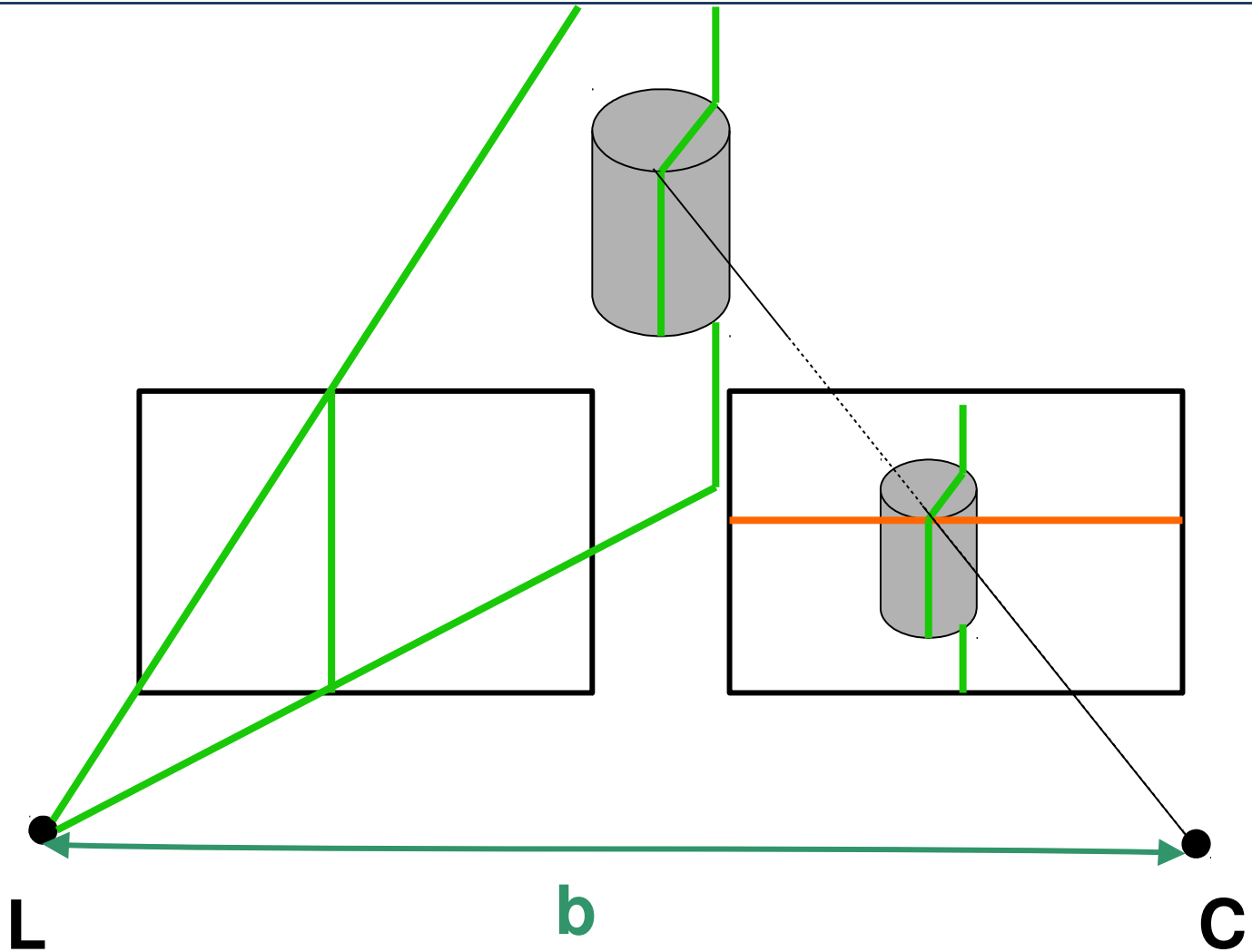




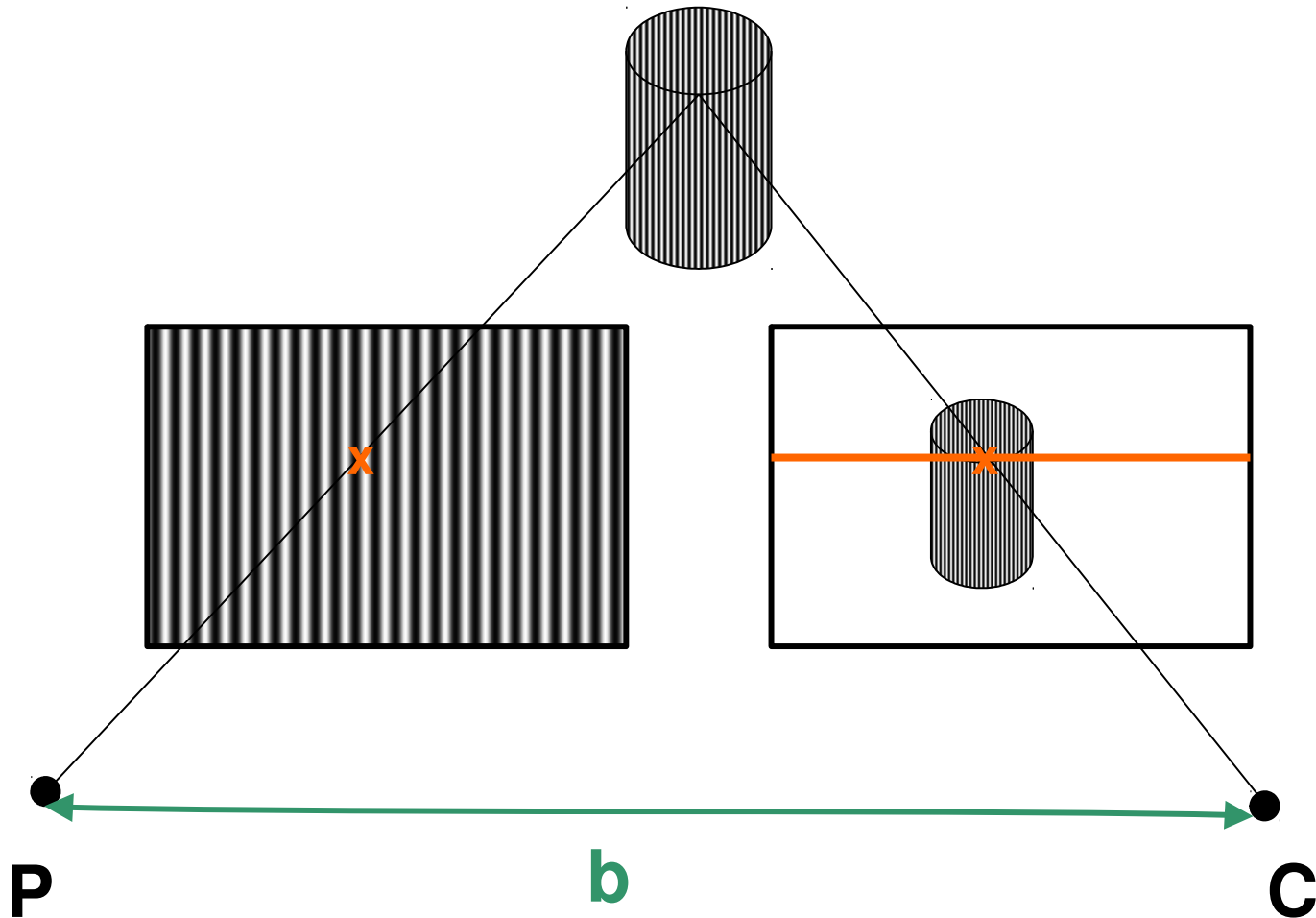
# Laser Scanner



# Laser Scanner



# Active Depth Estimation



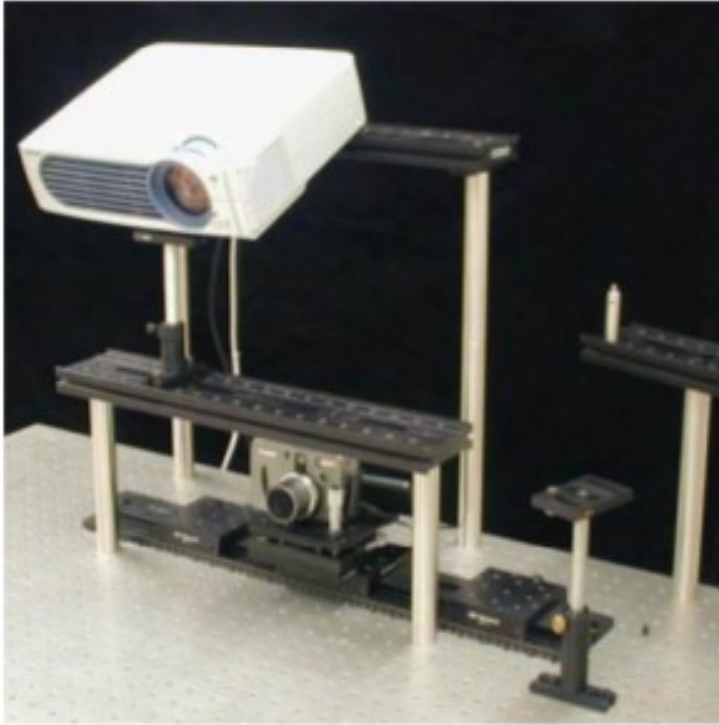
# Active Depth Estimation

- Advantages
  - Works on un-textured objects !!!
  - Works in dark environments
- Disadvantages
  - Sensible to incident radiation (e.g. sunlight)
  - Need to generate radiation

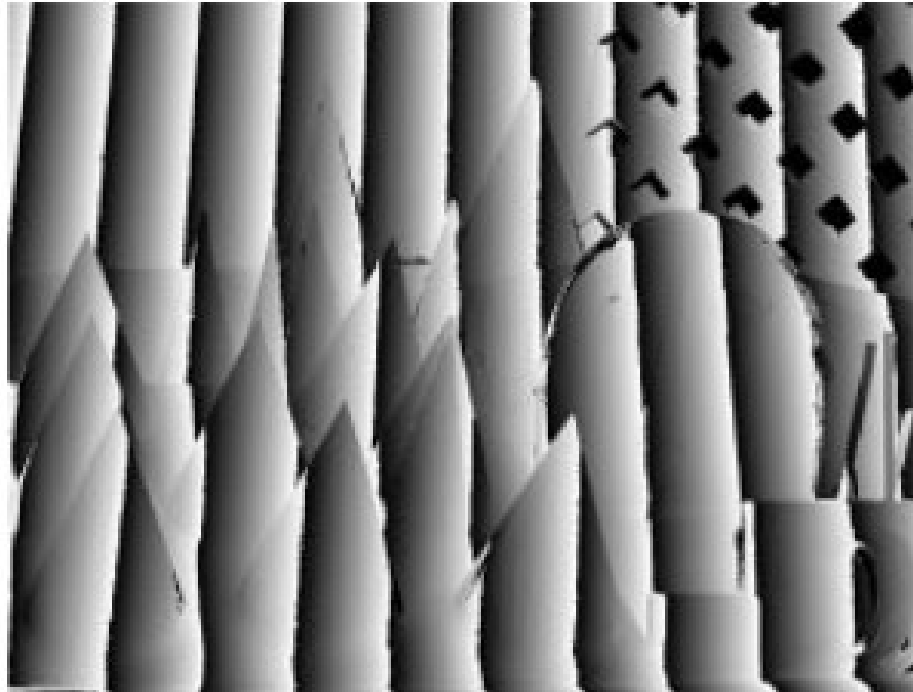
# Active Depth Estimation



# Active Depth Estimation



# Active Depth Estimation

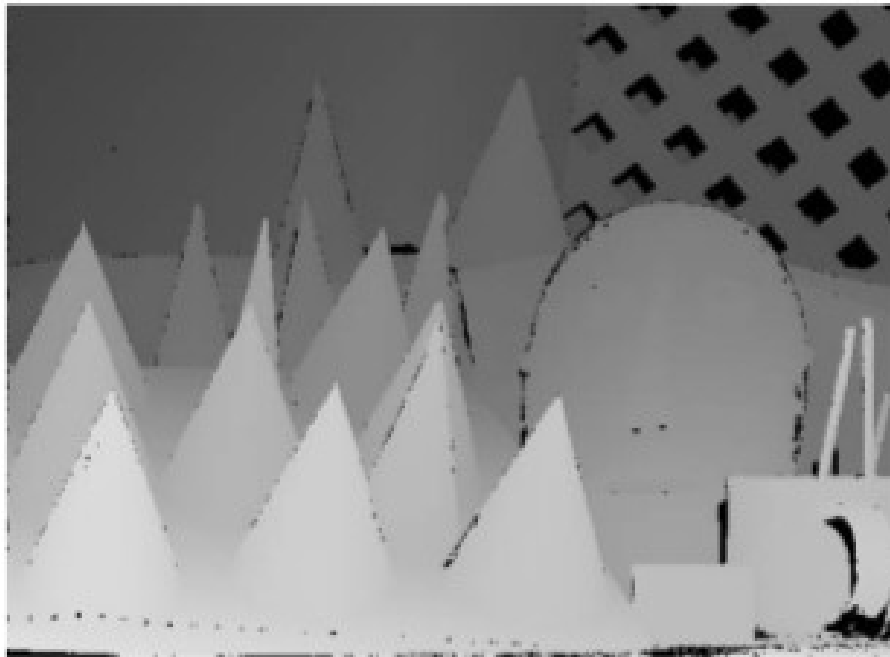


# Active Depth Estimation





# Active Depth Estimation



# Active Depth Estimation



# Active Depth Estimation



# Stereo Algorithm Benchmark

Set: [test dense](#) [test sparse](#) [training dense](#) [training sparse](#)

Metric: [bad 0.5](#) [bad 1.0](#) [bad 2.0](#) [bad 4.0](#) [avgerr](#) [rms](#) [A50](#) [A90](#) [A95](#) [A99](#) [time](#) [time/MP](#) [time/GD](#)

Mask: [nonocc](#) [all](#)

☐ plot selected ☐ show invalid [Reset sort](#)

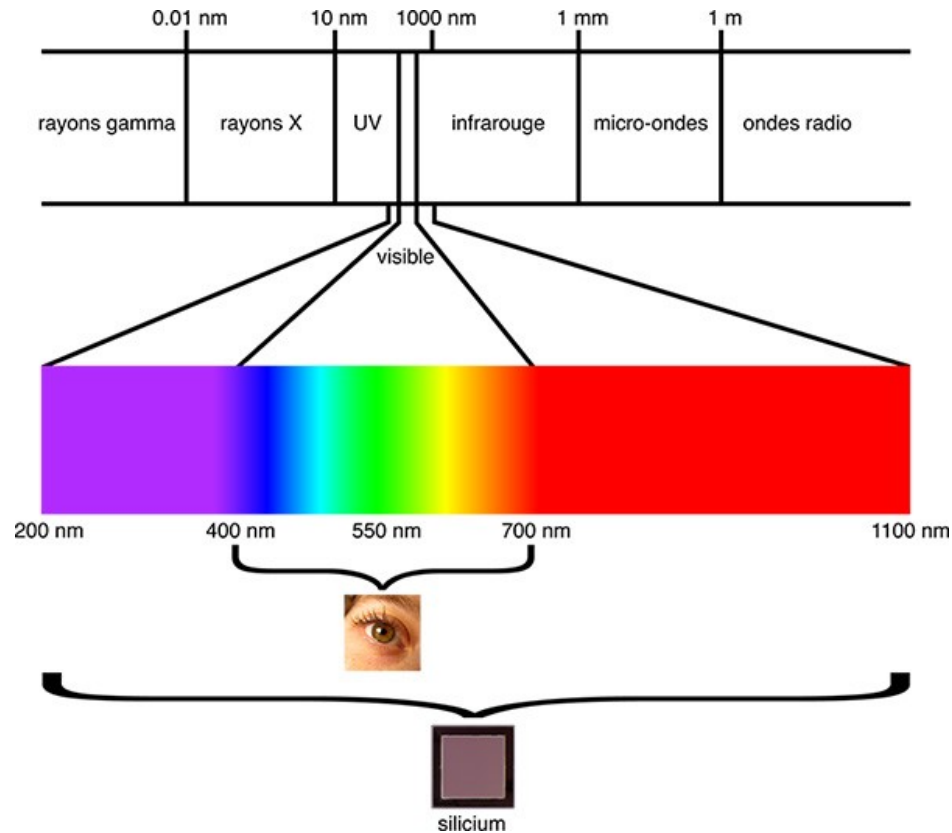
| Date     | bad 2.0 (%)              | Name        | Res | Weight | Avg     | Austr                                         | AustrP                                        | Bicyc2                                        | Class                                         | ClassE                                        | Compu                                         | Crusa                                         | CrusaP                                        | Djemba                                        | DjembaL                                       | Hoops                                         | Livgrm                                        | Nkuba                                         | Plants                                        | Stairs                                        |
|----------|--------------------------|-------------|-----|--------|---------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
|          |                          |             |     |        |         | MP: 5.6<br>nd: 290<br>im0 im1<br>GT<br>nonocc | MP: 5.6<br>nd: 290<br>im0 im1<br>GT<br>nonocc | MP: 5.6<br>nd: 250<br>im0 im1<br>GT<br>nonocc | MP: 5.7<br>nd: 610<br>im0 im1<br>GT<br>nonocc | MP: 5.7<br>nd: 610<br>im0 im1<br>GT<br>nonocc | MP: 1.5<br>nd: 256<br>im0 im1<br>GT<br>nonocc | MP: 5.5<br>nd: 800<br>im0 im1<br>GT<br>nonocc | MP: 5.5<br>nd: 800<br>im0 im1<br>GT<br>nonocc | MP: 5.7<br>nd: 320<br>im0 im1<br>GT<br>nonocc | MP: 5.7<br>nd: 320<br>im0 im1<br>GT<br>nonocc | MP: 5.7<br>nd: 410<br>im0 im1<br>GT<br>nonocc | MP: 5.9<br>nd: 320<br>im0 im1<br>GT<br>nonocc | MP: 5.5<br>nd: 570<br>im0 im1<br>GT<br>nonocc | MP: 5.6<br>nd: 320<br>im0 im1<br>GT<br>nonocc | MP: 5.2<br>nd: 450<br>im0 im1<br>GT<br>nonocc |
| ↕        |                          | ↕           | ↕   | ↕      |         | ↕                                             | ↕                                             | ↕                                             | ↕                                             | ↕                                             | ↕                                             | ↕                                             | ↕                                             | ↕                                             | ↕                                             | ↕                                             | ↕                                             | ↕                                             | ↕                                             | ↕                                             |
| 01/19/16 | <input type="checkbox"/> | NTDE        | H   |        | 7.62 1  | 5.72 2                                        | 4.36 4                                        | 5.92 1                                        | 2.83 1                                        | 10.4 1                                        | 8.02 1                                        | 5.30 1                                        | 5.54 1                                        | 2.40 1                                        | 13.5 1                                        | 14.1 1                                        | 12.6 2                                        | 13.9 3                                        | 6.39 1                                        | 12.2 3                                        |
| 08/28/15 | <input type="checkbox"/> | MC-CNN-acrt | H   |        | 8.29 2  | 5.59 1                                        | 4.55 7                                        | 5.96 2                                        | 2.83 1                                        | 11.4 4                                        | 8.44 3                                        | 8.32 2                                        | 8.89 4                                        | 2.71 2                                        | 16.3 3                                        | 14.1 2                                        | 13.2 4                                        | 13.0 1                                        | 6.40 2                                        | 11.1 2                                        |
| 11/03/15 | <input type="checkbox"/> | MC-CNN+FBS  | H   |        | 8.62 3  | 6.05 4                                        | 5.16 11                                       | 6.24 3                                        | 3.27 3                                        | 11.1 3                                        | 8.91 4                                        | 8.87 3                                        | 9.83 7                                        | 3.21 5                                        | 15.1 2                                        | 15.9 3                                        | 12.8 3                                        | 13.5 2                                        | 7.04 3                                        | 9.99 1                                        |
| 10/13/15 | <input type="checkbox"/> | MDP         | H   |        | 12.6 4  | 14.4 8                                        | 4.99 10                                       | 10.6 14                                       | 10.7 6                                        | 27.2 6                                        | 8.11 2                                        | 12.5 8                                        | 8.07 3                                        | 4.27 6                                        | 30.4 11                                       | 20.5 5                                        | 12.6 1                                        | 17.8 5                                        | 13.4 7                                        | 17.3 5                                        |
| 04/19/15 | <input type="checkbox"/> | MeshStereo  | H   |        | 13.4 5  | 5.90 3                                        | 4.88 8                                        | 10.8 15                                       | 12.9 11                                       | 10.6 2                                        | 13.6 8                                        | 12.2 7                                        | 9.01 5                                        | 5.39 10                                       | 27.4 5                                        | 23.5 8                                        | 17.7 6                                        | 21.0 12                                       | 15.4 11                                       | 20.9 8                                        |
| 11/06/15 | <input type="checkbox"/> | SOU4P-net   | H   |        | 13.5 6  | 23.1 12                                       | 5.41 13                                       | 6.39 4                                        | 13.1 12                                       | 30.5 10                                       | 11.1 5                                        | 16.4 11                                       | 12.7 12                                       | 3.13 3                                        | 28.9 7                                        | 17.1 4                                        | 16.4 5                                        | 16.9 4                                        | 10.7 5                                        | 14.5 4                                        |
| 12/18/15 | <input type="checkbox"/> | INTS        | H   |        | 14.8 7  | 20.2 10                                       | 4.52 5                                        | 8.62 10                                       | 11.6 7                                        | 29.5 8                                        | 13.7 9                                        | 16.4 10                                       | 10.3 9                                        | 4.69 7                                        | 27.6 6                                        | 22.5 6                                        | 20.7 9                                        | 20.5 10                                       | 11.5 6                                        | 24.9 11                                       |
| 11/05/15 | <input type="checkbox"/> | GCSVR       | H   |        | 14.8 8  | 17.1 9                                        | 3.50 1                                        | 8.22 9                                        | 16.5 17                                       | 47.4 17                                       | 11.4 6                                        | 9.75 5                                        | 7.06 2                                        | 3.17 4                                        | 34.4 15                                       | 27.1 10                                       | 18.3 7                                        | 19.2 8                                        | 16.0 12                                       | 19.3 7                                        |
| 11/12/14 | <input type="checkbox"/> | LCU         | Q   |        | 17.0 9  | 24.7 13                                       | 7.59 16                                       | 11.6 17                                       | 11.9 8                                        | 27.9 7                                        | 14.0 10                                       | 19.3 12                                       | 15.8 14                                       | 8.10 20                                       | 36.1 16                                       | 29.1 13                                       | 21.3 11                                       | 18.4 6                                        | 14.1 8                                        | 23.8 10                                       |
| 04/17/15 | <input type="checkbox"/> | TMAP        | H   |        | 17.1 10 | 20.2 11                                       | 4.94 9                                        | 8.13 8                                        | 12.8 10                                       | 30.0 9                                        | 14.1 12                                       | 27.9 17                                       | 20.4 18                                       | 5.09 8                                        | 31.5 13                                       | 23.1 7                                        | 20.9 10                                       | 19.0 7                                        | 18.8 16                                       | 18.0 6                                        |
| 10/31/15 | <input type="checkbox"/> | SPS         | F   |        | 18.2 11 | 12.1 7                                        | 11.4 22                                       | 13.3 21                                       | 12.1 9                                        | 15.7 5                                        | 17.0 21                                       | 14.0 9                                        | 14.3 13                                       | 8.31 21                                       | 30.3 10                                       | 32.3 15                                       | 30.0 26                                       | 25.2 21                                       | 23.7 21                                       | 26.2 14                                       |
| 10/07/14 | <input type="checkbox"/> | IDR         | H   |        | 18.4 12 | 37.5 20                                       | 4.08 2                                        | 7.49 6                                        | 23.3 20                                       | 40.6 13                                       | 15.7 19                                       | 24.5 13                                       | 11.3 11                                       | 5.46 12                                       | 33.1 14                                       | 26.0 9                                        | 21.5 12                                       | 21.7 13                                       | 15.3 10                                       | 21.2 9                                        |

# Scanning Humans



# Light Spectrum

- CCD chips have a broader range than eyes





# Near Infrared Imagery



[ Image thanks to D. Armstrong]

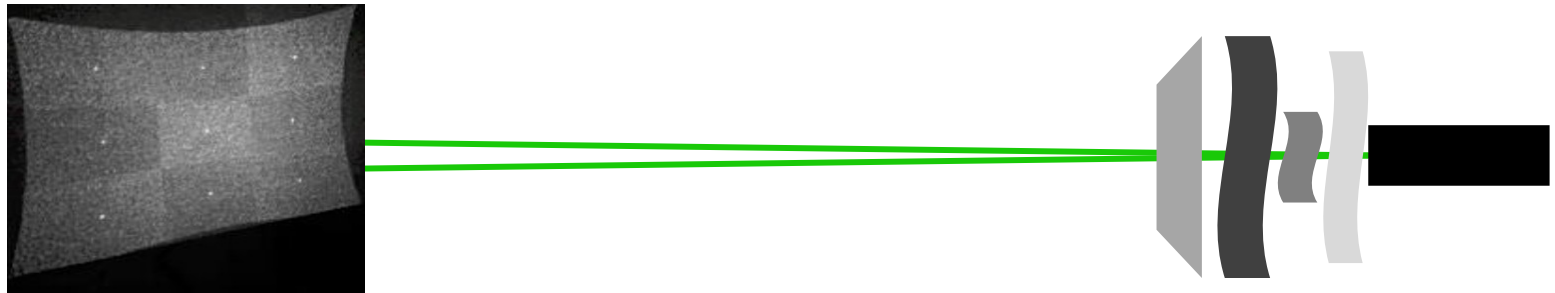
# The Hot Mirror





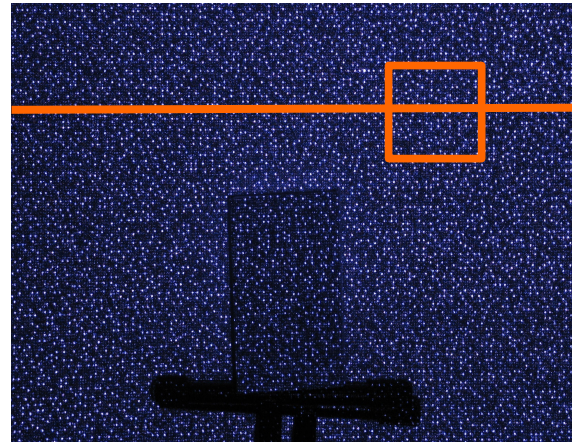
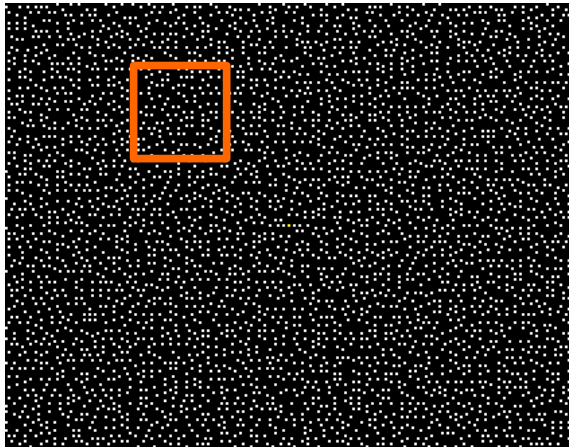
# Active Depth Cameras

- Laser light source with coherent light
- Ground glass as diffuser → Random speckle pattern
- Optical devices → Constant pattern



# Depth Estimation Algorithm

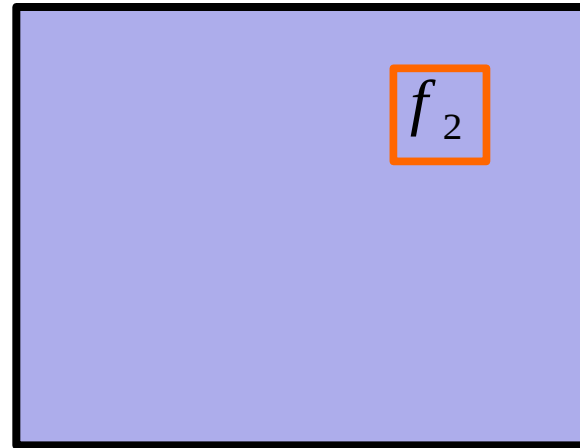
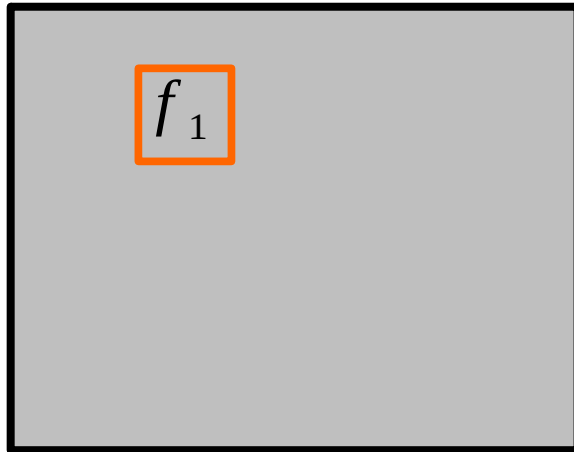
- Compare to reference pattern
- Normalized cross correlation on window (e.g. 16x16)



[R. Jain, K. Rangachar, B. Schunck (1995): *Machine vision*]

# Depth Estimation Algorithm

- Compare to reference pattern
- Normalized cross correlation on window (e.g. 16x16)



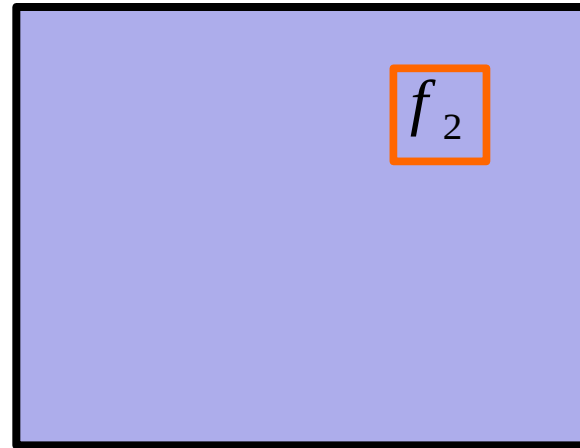
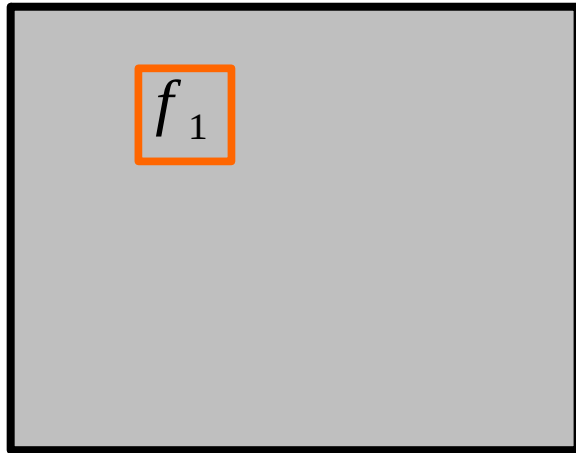
- Subtract mean  $F = f_1 - \bar{f}$   $G = f_2 - \bar{g}$
- Divide by variance  $\sigma_f = \sqrt{\sum_i F_i^2}$   $\sigma_g = \sqrt{\sum_i G_i^2}$

→ Invariant to brightness variations

[R. Jain, K. Rangachar, B. Schunck (1995): *Machine vision*]

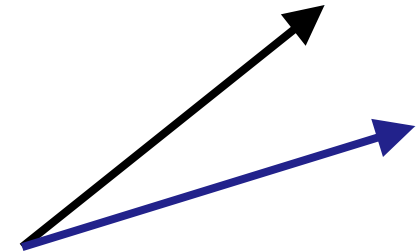
# Depth Estimation Algorithm

- Compare to reference pattern
- Normalized cross correlation on window (e.g. 16x16)

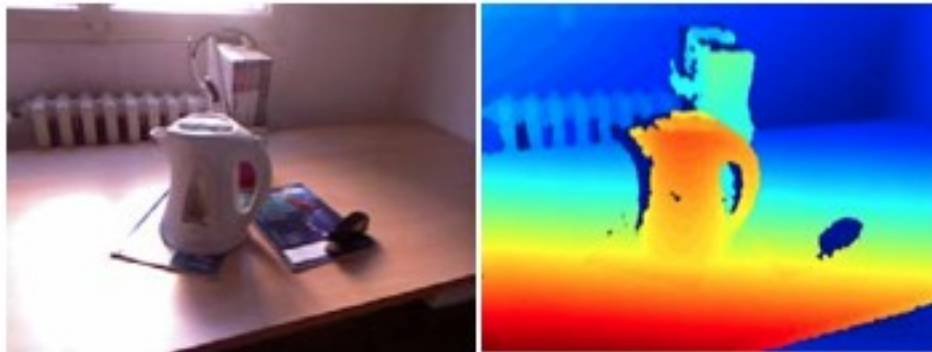


$$F = f_1 - \bar{f} \quad G = f_2 - \bar{g}$$
$$\sigma_f = \sqrt{\sum_i F_i^2} \quad \sigma_g = \sqrt{\sum_i G_i^2}$$
$$NCC = \frac{\sum_i F_i G_i}{\sigma_f \sigma_g}$$

$$NCC = \frac{F}{\|F\|} \frac{G}{\|G\|}$$



# Active Depth Estimation



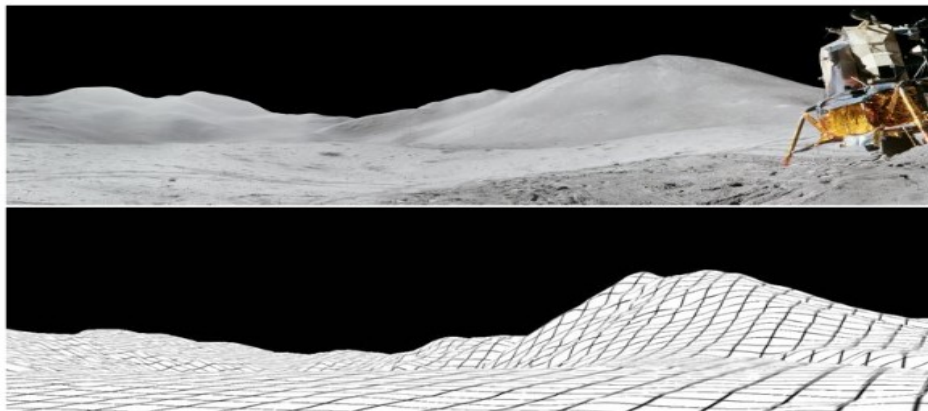
# Active Depth Estimation

- Challenges



# Depth Estimation with Images

- Stereo
- Active Stereo
- Time-of-flight
- Shape from shading
- Photometric stereo
- Depth from defocus



# Summary

- Computing the disparity field
  - Blockmatching
  - Dynamic programming
  - SGM
  - Belief Propagation
- Alternative depth estimation methods
- Additional Literature:
  - Bradski, Kaehler: Learning OpenCV
  - Ponce, Forsyth: Computer Vision, Chp. 11